Essays in Macroeconomics

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Chairman of the Doctoral Committee: Prof. Dr. Dieter Pfaff

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Preface

First and foremost, I want to thank my thesis supervisor Prof. Josef Zweimüller. Josef was my first lecturer in macroeconomics as a beginning undergraduate student and later engaged me as a research assistant, awaking my great interest in theoretical and empirical macroeconomic research. As a mentor and co-author, he helped me building the required knowledge and motivated me to pursue this research project. I am also very grateful to Prof. Reto Föllmi who taught me advanced theoretical macroeconomics, recruited me to the chair of Josef Zweimüller, and helped me refining my theoretical skills in numerous conversations as well as in our joint research projects. Special thanks also go to Prof. Josef Falkinger and to Prof. Fabrizio Zilibotti, my second supervisor, whose classes and seminars in macroeconomics have always been very inspiring. And I want to thank Prof. Ernst Fehr for enabling my visit to Cambridge, the MIT for hosting me, the SNF for financing part of my doctoral studies, and last but not least the faculty and support staff of the University of Zurich.

I want to thank my co-workers at the Department of Economics, especially at the chair of Josef Zweimüller. It was a great pleasure to work and study with so many bright, ambitious and fun colleagues. Being able to discuss research projects, class material or just the latest gossip has been immensely important to enhance my research and knowledge as well as to fuel my motivation. I also want to thank my friends outside (and inside) the sphere of economics for helping me relaxing from the hard work with the occasional beers.

Finally, I want to thank my mother, Barbara Würgler, who has always been a great supporter of my academic development, my father, René Würgler, who ignited my interest in economics, and my sisters, Tabea, Anja and Leonie. My last and deepest thanks go to the love of my life, Fabienne Piller.

Zurich, December 2010 Tobias Würgler

Chapter 1

Introduction

My thesis is centered on the macroeconomic causes and consequences of income inequality. The first two essays develop theoretical frameworks to analyze the impact of income inequality on technical change and long-term economic growth. These essays sprung from the very fruitful collaboration with my main thesis supervisor, Josef Zweimüller, who ignited my great interest in this topic. In the last essay I study the effect of financial booms and asset bubbles on wage inequality and sector employment both theoretically and empirically. This last topic was motivated by my previous work experience, allowing me to combine my applied knowledge accumulated in the financial sector with my modeling skills.

The first two essays analyze the impact of income inequality on the direction of technical change in models of endogenous growth with horizontal and vertical innovation. Traditionally, endogenous growth models assumed preferences admitting a representative household. If preferences are such that the composition and quality of consumption is identical across households, the income distribution does not matter for the aggregate consumption structure. In such an environment, models with process innovations (e.g. introducing new types of inputs) are mathematical identical to models with product innovations (see Acemoglu, 2009). The distinction between process innovations introducing higher quality versions of existing goods versus versions that can be manufactured at lower costs is less relevant, as well, if all households have the same willingness to pay for quality, which is implicitly assumed by admitting a representative household. However, casual observation and empirical evidence suggest that the consumption structure and willingness to pay for quality differs across households in reality, at odds with the representative household assumption. Richer households not only consume a larger variety of goods but also have a higher willingness to pay for quality upgrades. If the willingness to pay for variety and quality is determined by income levels, the income distribution affects prices and market sizes of firms and thereby incentives to invest in horizontal and vertical innovation differently. Consequently, the income distribution and the distinction between different types of innovations matter for aggregate outcomes.

The second chapter titled "The Macroeconomics of Model T" (Foellmi, Wuergler, and Zweimüller, 2009) studies a model of endogenous growth where firms invest both in product and process innovations. Product innovations introduce consumer goods which are affordable only to the rich. Process innovations decrease costs per unit of quality of the original making the good affordable to the poor. The automobile, one of the most important durable goods in modern industrial societies, provides a prototypical example for such an innovation cycle. Initially a luxury good consumed only by very rich households, things started to change in 1908, when Ford introduced the Model T, the car that "put America on wheels". The concept was the use of assembly lines to produce a low-cost, low-quality car affordable to the middle class. Model T became a huge success and initiated the takeoff in car ownership in the U.S. Motivated by this pattern which has also been observed for many other consumer durables, the essay develops a formal endogenous growth model where indivisibilities of consumption goods let the composition of demand by rich consumers systematically differ from that of poorer households. Income inequality thus shapes product cycles and generates substantially different incentives for product and process innovation. An egalitarian society creates incentives for process innovations (such as the Model T) whereas an unequal society favors product innovations (new luxuries). The inequality-growth relationship depends on the type of knowledge spillovers. The basic framework determines the fraction of mass producers but leaves individual product cycles indeterminate. The chapter also discusses natural extensions (learning-by-doing, hierarchic preferences) that generate deterministic product cycles where an initially exclusive good (which only the rich can afford) is subsequently transformed into a mass consumption good.

While the second chapter focuses on process innovations that cut manufacturing costs (at a quality discount), process innovations that introduce higher quality versions of a variety are as important in reality. The third chapter titled "Income Distribution and Product Quality versus Variety" (Wuergler, 2010a) explores the effects of income inequality on product quality and variety in a simple heterogeneous household economy. The income distribution is a key determinant of the quality levels and varieties produced and consumed in an economy when consumers' willingness to pay for quality and variety differs across levels of income. The chapter shows that product variety is unambiguously higher in an unequal society, whereas the quality level depends on the interaction between inequality and technology. If quality upgrades require high additional setup costs, the smaller markets of more unequal societies lower incentives for quality improvements. If in contrast quality upgrades entail high additional production costs, inequality increases quality by raising the willingness to pay for the manufacturing-intensive higher quality levels. In the presence of spillovers in vertical and horizontal R&D, a society may face a tradeoff between growth in quality and variety. Whereas an unequal society experiences higher growth in variety, growth in quality is higher in a more egalitarian society depending on technology. Finally, one can show that irrespective of technology, technical change is directed toward expanding variety if inequality is high, while it is biased toward improving quality in a more egalitarian society.

The final chapter is concerned with the causes rather than the consequences of income inequality. Titled "The Impact of Financial Booms on Labor Markets" (Wuergler, 2010b), the chapter studies the effect of financial booms and periods of extreme asset valuations on the relative demand for skills and the wage structure. The substantial rise in wage inequality in the U.S. since the late 1970s has been accompanied by a major expansion of financial services, a series of asset bubbles, and rising relative wages and relative education in the financial industry. I motivate and develop a theoretical framework where financial institutions benefit from financial booms and asset bubbles. Yet the complexity and novelty of financial products and fundamentals surrounding bubbles favor the supremacy of skilled individuals in exploiting these opportunities. Hence, financial booms increase opportunities for skilled labor, contributing to the rise in overall wage inequality in the economy. Simple extensions of the basic framework allow us to study the implications of financial regulation and globalization of financial services, as well as further topics. Finally, the chapter documents and compares relative wage and employment patterns in the U.S., U.K., Germany, and France, providing suggestive evidence for the theoretical framework.

Chapter 2

The Macroeconomics of Model T

Joint with Reto Foellmi and Josef Zweimüller

> "Consumer goods inventions that cut both cost and quality but reduce the former more than the latter, such as the Model T, have historically been an important means for transforming the luxuries of the rich into the conveniences of the poor."
>
> Jacob Schmookler, *Invention and Economic Growth* (1966)

2.1 Introduction

This chapter develops a model of endogenous growth based on a cycle of product and process innovations. Product innovations introduce new goods which are affordable only to the rich. Process innovations lead to the adoption of new production processes that reduce the cost per unit of quality, making the good affordable to the poorer classes. As emphasized by Schmookler (1966), such a cycle of product and process innovations has historically been important to transform the luxuries of the rich into mass consumption markets.

The automobile, one of the most important durable goods in modern industrial societies, provides a prototypical example for such an innovation cycle. In the United States, the history of the commercial automobile production started with Charles and Frank Duryea who founded the Duryea Motor Wagon Company in 1893, the first American automobile manufacturing company followed by Oldsmobile and Cadillac in 1902 and 1903. At the time, the automobile was a luxury good consumed only by very rich households. Things started to change in 1908, when Ford introduced the *Model T*, the car that "*put America on wheels*". The concept was the use of assembly lines to produce a low-cost, low-quality car affordable to the middle class. Model T became a huge success and initiated the takeoff in car ownership in the U.S. Between 1908 and 1927 more than 15 million units of Model T were manufactured. The introduction of Model T contributed crucially to the fast diffusion of the automobile in the U.S.[1]

Product cycles where a new invention created a luxury good for the rich and subsequent innovations turned the luxury into a mass consumption good for lower classes are not confined to the auto industry. It has been important for many other consumer durables such as the refrigerator, the radio, the TV, and the computer, showing very similar patterns of innovation cycles.

We develop a formal endogenous growth model where firms engage both in product and process innovations of indivisible consumption goods. These indivisibilities let the composition of demand by rich consumers systematically differ from that of poorer households. The rich do not only purchase a larger variety of consumption goods, but also do consume these goods in better quality. Poorer households consume only a fraction of the available varieties and prefer lower qualities to higher ones. Income inequality thus shapes product cycles and generates substantially different incentives for product and process innovation. Put differently, inequality determines the direction of technical change. Whereas an egalitarian society creates strong incentives for process innovations (such as the Model T), an unequal society creates strong incentives for product innovations (new luxuries).

Our analysis shows how the growth process depends on the extent of inequality in a society. *First*, the extent of inequality endogenously determines prices and market sizes. It turns out that higher inequalities allow innovators to charge higher prices and mark-ups, both for high- and for low-quality goods. Higher inequality reduces the number of mass markets due to two effects. The direct effect is that low incomes of the poor limit demand and the scope for mass production. The indirect effect comes from higher prices that limit the purchasing power of the poor even further. As a result, our analysis shows in a transparant way how inequality translates into price and market size effects. *Second,* prices and market sizes determine the incentives for product and process innovations. The way how these incentives eventually affect long-run growth depends on the

[1] Encyclopaedia Britannica

source of technical progress. If technical progress is mainly driven by product innovations, inequality is beneficial for long-run growth. In contrast, if technical progress is mainly driven by process innovations, the relationship between inequality and growth is turned upside down and inequality becomes harmful for long-run growth. In the presence of complementarities between process and product innovations, the relationship between inequality and growth becomes hump-shaped. Complementarities imply that an economy which has invested relatively little in process innovation is likely to benefit more from process innovations and vice versa. In that case, both very high levels and very low levels of inequality are harmful for growth, and growth is maximized at an intermediate extent of economic inequality.

In our basic model, we make the simplifying assumption that goods are symmetric. The general equilibrium determines the fraction of *exclusive producers* (that have incurred only the product innovation and supply only the high quality) and the fraction of *mass producers* (that have incurred both the product and process innovations cost and can supply both the high and the low quality). However, the product cycle of a particular variety is indeterminate. We show two natural ways to get rid of this indeterminacy and incorporate deterministic product cycles. A first way involves learning-by-doing where production experience lets production costs fall over time. Goods that are introduced earlier can be produced at lower cost, increasing innovators' incentive to open up mass markets. A second way is to assume hierarchic preferences where goods can be ranked according to priority in consumption (i.e. yield asymmetric utilities). In that case, product innovations follow the consumption hierarchy in the sense that product R&D expenditures are directed towards (not yet invented) goods with highest priority and process innovations are undertaken when the incomes of the poor have sufficiently increased. Both extensions generate a deterministic cycle with an initial phase of exclusion (only the rich can afford the new product) followed by the phase of mass consumption.

While the major part of our analysis studies the balanced growth path, we also explore transitional dynamics. Transitional dynamics reveal that both demand and supply shocks may trigger periods of industrial change in which a series of process innovations increases production and access to consumption markets. A large drop in inequality (such as the one that followed the Great Depression and WWII) triggers an initial phase where innovation activity is directed towards process innovations that facilitate mass production. Hence, our model provides an explanation for the boom in consumer durables in the U.S. (and other industrialized countries) in the post-war era. Similarly, a positive productivity shock lowering the costs of process innovation triggers an industrial revolution where an initially stagnant economy of craftsmanship and highly exclusive production is transformed into a modern society with broad participation and growth. We show that inequality – while initially beneficial for growth in the exclusive society – may eventually become harmful for growth after the economy has run through the transition phase and the economy has become a mass consumption society. In particular, our analysis predicts that in early stages of development (before the introduction of mass production technologies) inequality is beneficial for growth because technical progress is mainly driven by the introduction of new products for which the rich are willing to pay high prices. In later stages of development (after the introduction of mass production technologies) growth is higher in more egalitarian societies because process innovations become important drivers for growth. To generate the incentives for adopting these technologies, large markets and a high purchasing power of the lower classes are prerequisites.[2]

Our analysis extends the existing literature in at least three dimensions. *First*, our work is related to the literature on directed technical change (Acemoglu, 1998 and 2002, Acemoglu and Zilibotti, 2001, and others). This literature analyzes the forces that generate biases in technical change towards one particular production factor. Similar to our work, directed technical change models emphasize the tension between price and market size effects. However, the emphasis is on the relative demand for production factors, i.e. the supply/cost side of the economy. In contrast, our model focuses on demand/income effects. This channel generates an important role for the distribution of income across households, a mechanism that is absent in directed technical change models.

Second, our work highlights the distinct role that product and process innovations can play in the process of long-run growth. In this dimension we differ from the large literature on the determinants of the aggregate technical progress (Romer 1990, Aghion and Howitt, 1992, Grossman and Helpman 1991, etc.). Aggregate models of product and process innovations are often mathematically similar (Acemoglu, 2009), that is the source of technical change is not essential to answer the question of what factors influence economic growth. This is different in our framework where incentives for product inventions and process innovations are subject to systematic differences, in particular with respect to

[2] In Galor and Moav (2004, 2006) the inequality-growth relationship also changes across stages of development. Due to non-homothetic preferences over consumption and bequests, inequality leads to higher growth in early stages of development and to lower growth in later stages.

the extent of inequality in the society.

Third, we speak to a small literature that has studied the impact of income inequality on technical progress. Matsuyama (2002) demonstrates the virtuous cycle between learning-by-doing and a large middle class, enabling the Flying Geese pattern discussed later in the chapter. Foellmi and Zweimüller (2006) focus on product inventions and the scope of innovators' price setting power in the presence of a wealthy upper class. The present essay can be viewed as a synthesis of these classes of models. Our analysis highlights the conditions under which an unequal society suffers from lack of process innovations (and/or learning-by-doing) and from a small range of mass markets. Our analysis also makes precise the conditions under which such a society benefits from large mark-ups and high incentives to open up completely new product lines.[3]

The chapter is organized as follows: Section 2 analyzes empirical and historical evidence motivating the key assumptions and mechanisms of our model. Section 3 introduces the formal framework, section 4 presents the solution of the balanced growth equilibrium, and section 5 discusses the relationship between inequality and growth. Section 6 introduces alternative specifications of preferences and technology to allow for deterministic product cycles. Section 7 analyzes transitional dynamics. We conclude with a summary and potential directions for future research.

2.2 Motivating Evidence

Casual observations and empirical evidence suggest that there is a strong impact of income on the number of varieties purchased by households, which is at odds with homothetic preferences.[4] Figure 2.1 illustrates this point by exhibiting the shares of ownership of various consumer durables of urban Chinese households (National Bureau of Statistics of China). At any given point in time, most types of consumer durables are only consumed by a fraction of the households. The figure also shows that levels of penetration rise over time. This

[3]Murphy, Shleifer, and Vishny (1989) study the role of income distribution on technology adoption in a static context. Falkinger (1994) develops a model where inequality affects technical progress via aggregate output of consumer goods. The effect of inequality on technical progress in quality ladder models is explored in Li (2003) and Zweimueller and Brunner (2005).

[4]Jackson (1984) finds that the richest income class consumed twice as many different goods as the poorest class, using micro data from the Consumer Expenditure Survey of the Bureau of Labor Statistics. Falkinger and Zweimueller (1996) generate similar results using aggregate cross-country data from the International Comparison Project of the UN on per-capita expenditure levels on ninety-one different consumption categories.

is what Matsuyama (2002) calls the "Flying Geese pattern", in which a series of products takes off one after another, following an increase in productivity and income. This gradual increase in penetration levels was first emphasized by Katona (1964) who observed that the mass consumption society is the last stage of a process in which former luxury goods, consumed only by a few, privileged households, have been transformed into necessities for most households (i.e. mass consumption goods). Many products such as cars, radios, television sets, washing machines, refrigerators, vacuum cleaners and, more recently, computers have gone through such product cycles in the developed world, and are presently going through similar cycles in developing countries. Besides plain income effects, key elements of such product cycles are process innovations that cut the costs of production sufficiently. After a product has been invented, initial manufacturing costs are usually quite high, and sales volumes linger as the good can only be afforded by a few rich households. The takeoff and subsequent proliferation of the product is often ignited and enabled by a series of process innovations that reduce manufacturing costs significantly.[5]

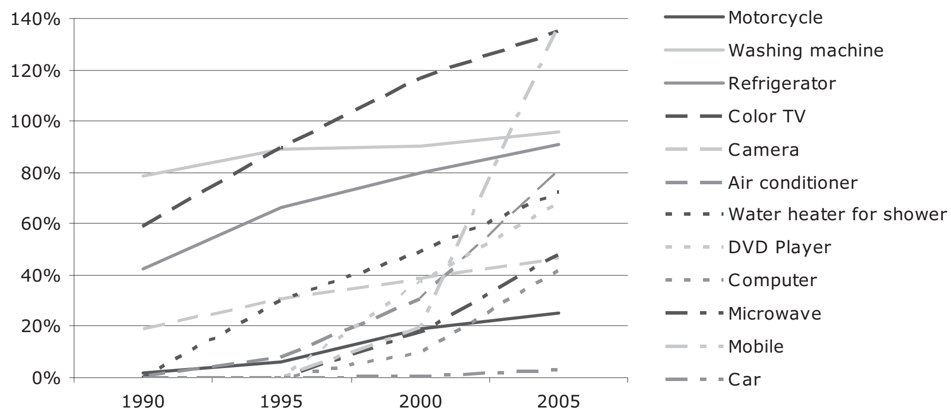

Figure 2.1: Ownership of consumer durables in Urban Chinese households (National Bureau of Statistics of China)

As mentioned above, one of the most famous historical examples for such

[5] Our analysis highlights the relevance of major product and process innovations that create new product lines and subsequent mass consumption goods. Notice that in reality both mass consumption goods and luxury goods are continuously improved in quality. While this is clearly of high relevance in practice, we abstract from continuous quality improvements in our framework.

an innovation pattern is the Ford Model T. It is generally regarded as the first affordable automobile, the car that "put America on wheels". One major reason behind the huge success story of Model T were Ford's innovations, including assembly line production instead of individual hand crafting, as well as the concept of paying the workers a wage proportionate to the cost of the car, so that they would provide a ready made market. Both innovations led to a huge increase in productivity. In total, Ford manufactured more than 15 million Model T's from 1908 to 1927, which contributed critically to the fast diffusion of the automobile. Figure 2.2 shows automobile and truck registrations in the U.S. from 1900 to 1970. The number of car registrations took off in the period of the Model T, and reached 23 million in 1927. Whereas 1% of households in the U.S. owned a car in 1908, the hour of birth of the Model T, penetration reached 50% in 1924.[6]

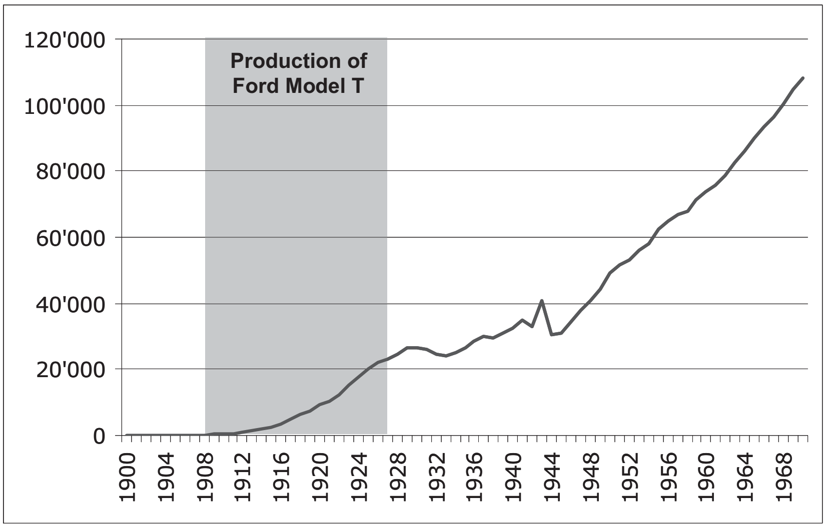

Figure 2.2: Automobile and truck registrations in the US in 1'000 units (US Census)

The product cycle that led to the Model T is not specific to the U.S. but can be observed in other parts of the world. Most of the large European economies had their own Model T which brought the car to the people. In Germany, a "people's car" – Volkswagen ("Beetle") – was initially introduced in the 1930s

[6] See Model T Facts on media.ford.com, Encyclopaedia Britannica, and Bowden and Offer (1994) for penetration levels.

(and fostered by the Nazi regime). Austin 7 (1922), Fiat (1936) and Citroën (1949)[7] brought the car to the people of the UK, Italy and France, respectively. In rich countries, the introduction of mass-produced cars was an important step in the history of the manufacturing industry. And what has been important for rich countries in the past is starting to become relevant in poorer countries today. In Asia for example, Tata has recently announced to produce the world's cheapest car, mainly for the Indian market.

The auto industry is an example for the types of innovation and product cycles that our model aims to capture. While it provided the prototypical example, there are many other goods that experienced very similar patterns of innovation and market expansion. Two centuries after artificial refrigeration was pioneered by Dr. William Cullen, a GE home *refrigerator* cost around 700$ in 1922, compared to 450$ for a 1922 Ford Model T. Penetration barely reached 1% in the U.S. in 1925. The introduction of freon expanded the refrigerator market during the 1930s, with penetration reaching 50% by 1938. Refrigerators went into mass production after WWII, and by the year 1948 75% of all households owned a fridge.[8] The history of *television* started with first experimental transmissions made by Charles Jenkins in 1923. Television usage in the U.S. exploded after WWII. Having reached a penetration of 1% in 1948, it only took 5 years to reach 50%, and 2 more years to reach 75%. The rapid diffusion was enabled by the lifting of the manufacturing freeze, war-related technological advances, the expansion of the television networks, the drop in television prices enabled by mass production and additional disposable income.[9] A very similar evolution can be traced for *computers*. Spurred by calculation requirements for ballistics and decryption during WWII, the first electronic digital computers were developed between 1940-1945. Developments of the microprocessor led to the proliferation of the personal computer after about 1975. Mass market pre-assembled computers allowed a wider range of people to use computers, and penetration reached 1% in the U.S. around 1980. Component prices continued to fall since then, leading to continuous price declines. Penetration reached 50% around 2000. The emergence of Netbooks in 2007, a new market segment of

[7] Citroën director Pierre-Jules Boulanger's early design brief for the 2CV supposedly asked for "a vehicle capable of transporting two peasants in boots, 100 pounds of potatoes or a barrel of wine, at a maximum speed of 40 mph, [...] Its price should be well below the one of our Traction Avant and, finally, its appearance is of little importance." (Translation, Technologie SCEREN - CNDP no. 138, 2005)

[8] Association of Home Appliance Manufacturers, "The Story of the Refrigerator;" Bowden and Offer (1994)

[9] Steven Schoenherr, "History of Television," History Server of University of San Diego; Bowden and Offer (1994)

small, energy-efficient ultra low-cost devices, is likely to advance penetration significantly, especially in developing countries.[10]

These examples demonstrate how closely process innovations and mass consumption markets are intertwined: Process innovations reducing manufacturing costs are crucial elements for tapping and proliferating mass consumption markets. Mass production, in turn, facilitates process innovation by increasing learning-by-doing and specialization benefits. Higher inequality raises the purchasing power of rich households, increasing demand for variety and product innovation. A more egalitarian society, on the other hand, raises the number of mass consumption markets and thus incentives for process innovation. Comparing the experience of Japan and the U.S. over the last decades provides suggestive evidence: Income concentration in Japan has remained relatively low after WWII in contrast to the U.S. (Moriguchi and Saez, 2005). During the same period of time, Japan has made itself a name as country of lean production and just-in-time management, i.e. process innovation. A recent study by Nagaoka and Walsh (2009), using data from the RIETI-Georgia Tech inventor survey, indeed shows that R&D in Japan is more biased to process innovation, in contrast to the U.S. where it is more directed to product innovation.

2.3 The Model

2.3.1 The Distribution of Endowments

We assume there are L households that inelastically supply L units of labor. $\beta L < L$ households are poor (indexed by P) and $(1-\beta)\,L$ are rich (indexed by R). Income differences arise from two sources. First, households are unequally endowed with units of labor. A poor household is endowed with $\ell_P = \theta_\ell < 1$ labor units, and the labor endowment of a rich household is $\ell_R = (1-\beta\theta_\ell)\,/\,(1-\beta) \geq 1$.[11] The parameters β and θ_ℓ fully characterize the distribution of labor endowments. The corresponding Lorenz-curve is piecewise linear with slope θ_ℓ for population shares between 0 and β; and slope $(1-\beta\theta_\ell)/(1-\beta)$ for population shares between β and 1. Notice that common measures of inequality (such as the Gini coefficient and the coefficient of variation) indicate an increase in inequality when θ_ℓ falls and/or β rises. It is

[10] Jeffrey Shallit, "A Very Brief History of Computer Science," University of Waterloo; W. Warner, "Great Moments in Microprocessor History," Technical Library IBM; "Computer Use and Ownership," U.S. Census, and authors' estimates

[11] Since the average labor endowment per household is unity we must have $\beta\ell_P+(1-\beta)\ell_R = 1$. Setting $\ell_P = \theta_\ell$ we get $\ell_R = (1-\beta\theta_\ell)/(1-\beta)$.

assumed that the distribution of labor endowments is constant over time.

The second source of income differences is due to inequality in wealth, based on ownership in monopolistic firms. We denote by $v(t)$ the per-capita value of these firms at date t and assume that a poor household owns wealth $v_P(t) = \theta_v(t)v(t)$ and a rich household owns wealth $v_R(t) = [(1-\beta\theta_v(t))/(1-\beta)]\, v(t)$ where $\theta_v(t) < 1$ and $(1-\beta\theta_v(t))/(1-\beta) \geq 1$. In analogy to the labor endowment distribution, the distribution of wealth is determined by β and $\theta_v(t)$. Unlike the labor endowment distribution, the wealth distribution can change over time since $v_P(t)$ and $v_R(t)$ are endogenously determined by households' savings decisions. In sections 4 to 6 below we will study balanced growth paths. Along such paths, all households have the same savings rates and the wealth distribution is stationary, $\theta_v(t) = \theta_v$ for all t. When we analyze balanced growth paths below we will assume $\theta_\ell = \theta_v = \theta$. While this is clearly a rather special case, it keeps the analysis simple and transparent. Allowing labor endowment and wealth distributions to differ does not change the results in any economically relevant way. For instance, in comparing steady states, it does not make a difference whether the resulting incomes differences arise due to an unequal labor endowment distribution, due to an unequal wealth distribution, or both. What matters is inequality in total lifetime incomes. However, when we study transitional dynamics in section 7, we have to account for the fact that households' savings rates need no longer be equal in the transition to a new steady state. As the wealth distribution changes over time we have to abandon the assumption $\theta_\ell = \theta_v = \theta$ and make the time-dependence of $\theta_v(t)$ explicit.

2.3.2 Technology and Technical Progress

Labor is the only production factor, the labor market is competitive and the market clearing wage is denoted by $w(t)$. Production activities are undertaken in monopolistic firms that supply differentiated products and operate with an increasing returns-to-scale technology. The creation of a firm requires a *product innovation*, i.e. an investment of $\tilde{F}(t)$ units of labor that yields the blueprint for a completely new product (e.g. the automobile). Once such a product innovation has been made, the innovating firm obtains a patent of infinite length granting the exclusive right to market this product. We think of a product innovation as a luxury good that initially may be affordable only to the rich and that is costly in production. We assume a new product has quality q_h and requires a (high) labor input $\tilde{a}_h(t)$ per unit of output. After a successful product innovation, the firm has the option to undertake a *process innovation* that cuts both the quality of the product and its production cost. More precisely, we assume that after a further investment of $\tilde{G}(t)$ labor units, the product can also be supplied in lower quality $q_l < q_h$ and produced with a lower labor input $\tilde{a}_l(t) < \tilde{a}_h(t)$, the quality-cost ratio is higher, however, $q_l/\tilde{a}_l(t) > q_h/\tilde{a}_h(t)$. This captures Schmookler's idea that mass consumer good inventions cut both costs and quality but the former more than the latter.[12]

In what follows we will refer to firms that have incurred both the product and the process innovation as "*mass producers*". Firms that have made only the product but not the process innovation will be called "*exclusive producers*". The term "exclusive" is suggestive in the sense that it refers to both a high "exclusive" quality and to a situation where firms "exclude" the poor from consumption by setting prices that only rich but not poor households can afford.[13]

Product and process innovations are the driving forces behind technical progress and long-run growth. Sustained growth is enabled by knowledge spillovers from past research activities on current productivity levels.[14] Labor requirements in the various activities are inversely related to an aggregate stock of knowledge $A(t)$ such that $\tilde{F}(t) = F/A(t)$, $\tilde{a}_h(t) = a_h/A(t)$, $\tilde{G}(t) = G/A(t)$, and $\tilde{a}_l(t) = a_l/A(t)$ where F, G, a_h, and a_l are exogenous, positive constants. Balanced growth requires the stock of knowledge $A(t)$ to be a linearly homogeneous function of the range of product varieties $N(t)$ and the range of varieties that underwent process innovations $M(t)$. For analytical convenience, we assume that $A(t)$ is linked to past product and process innovations via the CES-function

$$A(t) = \left[\psi N(t)^{\gamma} + (1-\psi) M(t)^{\gamma}\right]^{1/\gamma}, \tag{2.1}$$

[12] Note that we abstract from continuous quality improvements of existing goods which are important features of reality. The model could be easily adapted to include exogenous quality improvements. If q_h and q_l increased at an exogenous rate, all features of our model would remain the same. We will also touch upon quality improvements when discussing learning-by-doing in Section 6. Furthermore, both high- and low-quality versions of a variety are produced by one firm as a first approximation. In reality, process innovations are often undertaken by competitors to enter an existing product line. Hence, one could extend the present model to a duopolist setting to study the competitive effects of the process innovator on the original product inventor.

[13] Note that the way we use the terms "exclusive producers" and "mass producers" refers to access to technology rather than to quantity of production. It may be that a mass producer makes a higher profit by selling only to the rich and a luxury producer may be better off by selling to the rich and the poor. We will see that such "strange" outcomes never happen along a balanced growth path but may be temporarily relevant during transitions towards a new steady state (see section 6 below).

[14] The knowledge-driven specification is more simple and transparent in a setting with consumer good varieties of different quality as opposed to the lab equipment R&D specification.

where $\gamma < 1$ parametrizes the substitutability between experience in product and process innovations, and $\psi \in [0, 1]$ the importance of product relative to process innovations for knowledge accumulation. If ψ is high, technical progress and growth are mainly driven by experience accumulated in product R&D. If it is low, process innovations are the main driver. The lower γ, the more complementary product and process innovations are. Note that both R&D sectors benefit equally from spillovers. An extension of the model could study spillovers affecting product and process innovation differently, which will be discussed when we present the main results. We will show that in an equilibrium where only the producers who choose to sell to all households have invested in process innovation, the inequality-growth relationship depends on the specific form of spillovers. In order to highlight that our basic results and intuitions do not necessarily rely on specific externalities between product and process innovations, Section 6 studies an alternative setup in which manufacturing costs are lowered through learning-by-doing instead of intentional process innovations.

2.3.3 Preferences and Consumer Choices

Households have an infinite horizon and choose consumption both within and across periods to maximize lifetime utility. At a given point in time, a household chooses consumption from the continuum of $N(t)$ goods. Among the $N(t)$ firms that exist at date t there are those that made a product innovation but have not yet made a process innovation (exclusive producers) and other firms that have made both the product and the process innovation (mass producers). This means $M(t)$ goods are supplied both in high and low quality and $N(t) - M(t)$ goods are supplied in high quality only. In general, the prices may vary both across goods and across qualities and may change over time. We denote the price of good j and quality q at date t by $p(j, q, t)$.

The crucial assumption adopted here is that goods are indivisible. More precisely, the household has to decide whether or not to consume good j, and if yes, whether to consume it in high or low quality. There are three outcomes: either a household consumes (i) one unit in high quality, (ii) one unit in low quality, or (iii) does not consume at all. It turns out that such a discrete specification of preferences is a simple and tractable way to introduce non-homotheticities and to allow for a situation where rich households do not only consume a broader menu of goods but also consume the purchased goods in higher quality. Denote by $x_i(j, t)$ an indicator function that takes value 1 if household i consumes good j at date t, and takes value 0 if not. Similarly, denote by $q_i(j,t)$ the chosen quality level which can take only one of the two values $\{q_h, q_l\}$. The household's objective function is given by

$$U_i(\tau) = \int_\tau^\infty \frac{1}{1-\sigma}\left[\int_0^{N(t)} x_i(j,t)q_i(j,t)dj\right]^{1-\sigma} e^{-\rho(t-\tau)}\ dt,$$

where ρ is the rate of time preference, and σ parametrizes the willingness to shift consumption across time. The term in brackets can be interpreted as an instantaneous consumption aggregator which, for later use, we denote by $c_i(t) \equiv \int_0^{N(t)} x_i(j,t)q_i(j,t)dj$. The consumer chooses the time paths of $x_i(j,t)$ and $q_i(j,t)$ so as to maximize the above lifetime utility subject to the lifetime budget constraint

$$\int_\tau^\infty \left[\int_0^{N(t)} p(j,q_i,t)x_i(j,t)dj\right] e^{-R(t,\tau)}dt \leq \int_\tau^\infty \ell_i w(t)e^{-R(t,\tau)}dt + v_i(\tau),$$

where $R(t,\tau) = \int_\tau^t r(s)ds$ is the cumulative discount factor between dates τ and t, $r(t)$ is the interest rate, ℓ_i is the (time-invariant) labor endowment of household i, and $v_i(\tau)$ is the initial wealth level owned by the household.

The first-order conditions for the discrete consumption choice of good j are given by

$$\{x_i(j,t), q_i(j,t)\} = \begin{cases} \{1, q_h\} & \text{if} \quad q_h\mu_i(t) - p(j,q_h,t) \geq \max\left[0, q_l\mu_i(t) - p(j,q_l,t)\right], \\ \{1, q_l\} & \text{if} \quad q_l\mu_i(t) - p(j,q_l,t) \geq \max\left[0, q_h\mu_i(t) - p(j,q_h,t)\right], \\ \{0, \cdot\} & \text{otherwise,} \end{cases} \tag{2.2}$$

where

$$\mu_i(t) = c_i(t)^{-\sigma}/\lambda_i(t)$$

is household i's willingness to pay per unit of quality and $\lambda_i(t)$ the marginal utility of wealth at date t (the current-value multiplier). These first order conditions are very intuitive. The condition in the first line of (2.2) says that good j will be consumed in high quality if the consumer's willingness to pay for the high quality $q_h\mu_i(t)$ is sufficiently larger than its price $p(j,q_h,t)$ so that both alternatives (purchasing not at all and purchasing the low quality) lead to a worse outcome. In other words, there needs to be a utility gain and it needs to be larger than the utility gain from purchasing the low quality. Similarly, the consumer will purchase the low quality if there is a utility gain that is larger than when purchasing the high quality. Otherwise, the household does not consume good j at all.

2.3.4 Price Setting and Profits

Firms make their pricing decisions on the basis of market demand functions that derive from households' optimal consumption choices given by the conditions in (2.2). Notice that the willingness to pay for quality $k \in \{l, h\}$ is always larger for a rich household than for a poor household, $q_k\mu_R > q_k\mu_P$. (For simplicity, we omit time indices in this section).

An *exclusive producer* can supply the product only in high. When the firm charges a price below (or equal to) $q_h\mu_P$ both rich and poor households will purchase the good and market demand is L. When the price is above $q_h\mu_P$ but below (or equal to) $q_h\mu_R$ only rich households purchase the good and market demand is $(1-\beta)L$. When the price is larger than $q_h\mu_R$ not even the rich are willing to purchase and market demand is zero. The exclusive producer has essentially two options: (i) set price $q_h\mu_R$ and sell to rich households only; or (ii) set price $q_h\mu_P$ and sell to the whole customer base.

A *mass producer* can supply the good both in high and low quality. The mass producer has in principle the following options: (i) supply only the low quality at price $q_l\mu_P$ to all households; (ii) supply the low quality at price $q_l\mu_R$ only to rich households; (iii) supply the high quality at price $q_h\mu_R$ only to rich households; or (iv) supply the high quality at price $q_h\mu_P$ to all households. Actually, the mass producer has a fifth option and this option is the most interesting one in the present context: (v) set price $q_l\mu_P$ for the low quality and sell it to poor households and set price $q_l\mu_P + (q_h - q_l)\,\mu_R$ for the high quality and sell it to rich households. Notice that under this fifth option the firm cannot fully exploit the willingness to pay of rich consumers since they can switch to the low quality. To attract the rich households as customers for the high quality, the firm needs to set a price that is not larger than the price that makes a rich household indifferent between consuming the low quality and consuming the high quality. From (2.2) it is straightforward to verify that, when the low quality has price $q_l\mu_P$, the highest price that induces the rich to purchase the high rather than the low quality is $q_l\mu_P + (q_h - q_l)\,\mu_R$. To ensure that in equilibrium a situation emerges, where a mass producer sells the high quality to the rich and the low quality to the poor, we make the following assumption:

The following three conditions are satisfied: (i) $(q_h - q_l)\,\mu_R > (\tilde{a}_h - \tilde{a}_l)\,w$, (ii) $(1-\beta)\,(q_h\mu_R - \tilde{a}_h w) \geq (q_h\mu_P - \tilde{a}_h w)$, and (iii) $q_l\mu_P - (1-\beta)\,q_l\mu_R - \beta\tilde{a}_l w \geq 0$, where μ_R and μ_P are determined by equations (2.6) - (2.9).

The willingnesses to pay of rich and poor households, μ_R and μ_P, will be determined endogenously in general equilibrium (see next section). Condition (i) says that the willingness to pay of rich households for the quality gap $q_h - q_l$

is sufficiently high relative to the cost gap $(\tilde{a}_h - \tilde{a}_l)\, w$ so that a mass producer strictly prefers selling the high quality to the rich and the low quality to the poor at prices $q_l\mu_P$ and $q_l\mu_P + (q_h - q_l)\mu_R$, respectively, to selling the low quality at price $q_l\mu_P$ to all consumers. Condition (ii) says that an exclusive firm weakly prefers selling only to rich households at price $q_h\mu_R$ rather than selling to all households at price $q_h\mu_P$. Condition (iii) says that a producer with access to the mass production technology is weakly better off separating the market (selling the low quality to the poor and the high quality to the rich) rather than selling the high quality only to the rich at a higher price $q_h\mu_R$. Our assumption $q_l/\tilde{a}_l > q_h/\tilde{a}_h$ guarantees that (ii) and (iii) are compatible.

In the next sections we study the balanced growth path where all exclusive producers sell their high quality only to the rich, and all mass producers sell the low quality to the poor and the high quality to the rich. Along this path all inequalities in Assumption 1 hold strictly. This does not need to be the case during a transition towards the balanced growth path. The case where condition (ii) holds with equality and condition (iii) holds with strict inequality corresponds to a situation where the economy has few mass producers, so that the poor purchase all mass consumption goods in low quality but also purchase some luxuries. The case where condition (iii) holds with equality and condition (ii) holds with strict inequality corresponds to a situation where there are so many mass producers that the poor cannot afford to purchase all mass consumption goods but only a subset of them.[15]

Proposition 1 *a) Suppose conditions (ii) and (iii) in Assumption 1 hold with strict inequality. Then every exclusive producer sells only to the rich, charges price* $p_e = q_h\mu_R$ *and earns profit* $\pi_e = (1-\beta)\, L\, (p_e - \tilde{a}_h w)$. *Every mass producer sells the low quality to the poor at price* $p_l = q_l\mu_P$ *and the high quality to the rich at price* $p_h = q_l\mu_P + (q_h - q_l)\,\mu_R$ *and earns profit* $\pi_m = (1-\beta)\, L\, (p_h - \tilde{a}_h w) + \beta L\, (p_l - \tilde{a}_l w)$. *b) When condition (ii) holds with equality, exclusive firms are indifferent between selling only to rich and to all households. c) When condition (iii) holds with equality, mass firms are indifferent between selling the high quality only to the rich at price* $q_h\mu_R$ *and separating the market. In that case we have* $\pi_e = \pi_m$.

Proof. *See Appendix A.* ■

[15] The assumption $q_l/\tilde{a}_l > q_h/\tilde{a}_h$ precludes that both (ii) and (iii) hold with equality. Also notice that the rich purchase all goods in every case. Both exclusive and mass producers which do not sell to the rich have strictly lower profits and hence will undercut prices to get the rich as customers. Similarly, firms that sell to some poor households sell to all poor households.

It is also instructive to see what happens if some of the conditions of Assumption 1 are violated. In that case, mass producers supply only one quality. They may sell only the low quality to the whole customer base. This case is similar to the one we will study below. Alternatively, mass producers may not have an incentive to supply the low quality. This case is obviously not interesting because there does not exist an incentive to undertake a process innovation and the model essentially reduces to one of expanding product varieties.[16]

2.3.5 R&D and Resources

Inventing a new good and setting up a new exclusive firm is attractive as long as the value of this product innovation (the present value of future cash flows) does not fall short of the initial R&D cost. Initial R&D costs are $w(t)\tilde{F}(t)$ and, taking labor as the numeraire so that $w(t) = A(t)$, we have $w(t)\tilde{F}(t) = F$. The present value of a new innovation depends on whether and, if so, when the firm implements the mass production technology. The process innovation costs are $w(t)\tilde{G}(t) = G$. Denote by $\Delta(j)$ the duration between the product innovation and the process innovation, i.e. the firm "age" at which to implement the mass production technology; and by $\pi_e(j,t)$ and $\pi_m(j,t)$ the profits before and after implementing mass production, respectively. Then the value of a firm that introduces a new product at date τ is given by

$$V(j,\tau) = \max_{\Delta(j,\tau)} \left[\int_{\tau}^{\tau+\Delta(j)} \pi_e(j,t) e^{-R(t,\tau)} dt + \int_{\tau+\Delta(j)}^{\infty} \pi_m(j,t) e^{-R(t,\tau)} dt - G e^{-R(\tau+\Delta(j),\tau)} \right],$$

With free entry into the R&D sector, the general equilibrium leaves no profit opportunities unexploited. Hence the value of a product innovation cannot exceed the initial R&D cost $V(j,t) \leq F$.

Finally, the economy-wide resource constraint has to be satisfied at all times. Aggregate labor supply is fixed to L. Aggregate labor demand comes from the R&D sector and the production sector which produces (high- and low-quality) output. In the R&D sector, $\dot{N}(t)\tilde{F}(t)$ units of labor are engaged in designing entirely new products, and $\dot{M}(t)\tilde{G}(t)$ units of labor are used to implement new mass production technologies. In the production sector $Y_h(t)a_h(t)$ and $Y_l(t)a_l(t)$

[16] In the dynamic context this means there is no incentive to undertake a process innovation because the return to this investment is too low. An alternative polar case would be one where firms have an extremely high incentive to undertake the process innovation because process innovations are very cheap. In that case all firms would invest in both product and process innovation right from the beginning, again reducing the framework to a situation of expanding product varieties in which the high quality is never produced. See Appendix C.

units of labor are employed to produce high-quality and low-quality output denoted by $Y_h(t)$ and $Y_l(t)$, respectively. The resource constraint of the economy can be written as

$$Y_h(t)\tilde{a}_h(t) + Y_l(t)\tilde{a}_l(t) + \dot{N}(t)\tilde{F}(t) + \dot{M}(t)\tilde{G}(t) \leq L.$$

2.4 General Equilibrium and Balanced Growth

We are now ready to consider the dynamic general equilibrium of the economy described above. In the next sections we analyze the balanced growth path and leave the analysis of transitional dynamics to section 7 below. In the balanced growth equilibrium, there is both continuous introduction of entirely new products and continuous adoption of new processes that allow mass production of former exclusive goods. In the main text we focus on the most interesting equilibrium situation where mass producers sell the high quality to the rich and the low quality to the poor, i.e. where Assumption 1 holds. Situations where Assumption 1 does not hold are analyzed in Appendix C.

Definition 1 *A* balanced growth equilibrium *in our economy consists of a path where the interest rate $r(t)$ is constant; the stock of knowledge $A(t)$, the wage rate $w(t)$, the total number of firms $N(t)$, and the number of mass producers $M(t)$ grow at the constant rate g. Hence the fraction of mass producers $m = M(t)/N(t)$ is constant and labor requirements $\tilde{a}_h(t)$, $\tilde{a}_l(t)$, $\tilde{F}(t)$, and $\tilde{G}(t)$ shrink at rate g. Profit maximizing prices $p_e(j,t)$, $p_h(j,t)$ and $p_l(j,t)$, and instantaneous profits $\pi_e(j,t)$ and $\pi_m(j,t)$ are the same for all firms and constant over time. Given Assumption 1, rich households consume all $N(t)$ goods in high quality and poor households consume all $M(t)$ mass consumption goods in low quality. Hence the level of consumption of rich $c_R(t) = q_h N(t)$ and poor $c_P(t) = q_l M(t)$ also grows at rate g. Both types of households have the same savings rate, so the distribution of wealth is stationary.*

2.4.1 Product and Process Innovations

In a balanced growth equilibrium, the profits of exclusive and mass producers are constant over time and given by π_e and π_m defined in Proposition 1 and the interest rate r is constant. The optimal timing of the process innovation simplifies to

$$\max_{\Delta} \int_{\tau}^{\tau+\Delta} \pi_e e^{-r(t-\tau)} dt + \int_{\tau+\Delta}^{\infty} \pi_m e^{-r(t-\tau)} dt - G e^{-r\Delta}.$$

Using the Leibniz rule we obtain

$$\Delta = \begin{cases} 0 & \text{if} \quad (\pi_m - \pi_e)/r > G, \\ [0,\infty) & \text{if} \quad (\pi_m - \pi_e)/r = G, \\ \infty & \text{if} \quad (\pi_m - \pi_e)/r < G. \end{cases}$$

The above condition says that the present value of the increased profit flow is compared to innovation costs. We are interested in an equilibrium outcome where exclusive producers and mass producers co-exist so the first and third case of the above condition can be ruled out. This means the optimal timing of a process innovation Δ is undetermined. In other words firms are indifferent whether and when to invest in process innovation. However, the aggregate fraction of firms which have invested in process innovation, i.e. the fraction of mass producers m, is determined in equilibrium. The indeterminacy of the individual product cycle is due to the symmetry in preferences and technology. The symmetry assumption is not critical for our results. In fact, introducing asymmetries in our basic framework generates deterministic product cycles featuring the empirically observed patterns mentioned in Section 2. Section 6 analyzes two such extensions.

Returning to the basic model, the following no-arbitrage conditions must hold:

$$\begin{aligned} V_N = \tfrac{\pi_e}{r} &= \tfrac{(1-\beta)L(q_h\mu_R - a_h)}{r} = F, \\ V_M = \tfrac{(\pi_m - \pi_e)}{r} &= \tfrac{L[q_l\mu_P - (1-\beta)q_l\mu_R - \beta a_l]}{r} = G. \end{aligned} \tag{2.3}$$

Note that, along the balanced growth path, all involved variables are constant over time. The present value of the profit flow enabled by product innovation V_N must be equal to initial product R&D costs. And the present value of the incremental profit flow enabled by subsequent process innovation V_M must be equal to process innovation costs. Note also that V_N increases in the purchasing power of the rich, while V_M increases in the purchasing power of the poor. Higher inequality raises incentives for product innovation relative to process innovation, while a more egalitarian society increases incentives for process innovation.

2.4.2 Growth and Mass Production

In a balanced growth equilibrium, expenditures grow at rate g and prices are constant. Hence, consumption growth of poor and rich households follows the standard Euler equation:

$$r = \sigma g + \rho, \tag{2.4}$$

Because poor households are endowed with θ units of labor and $\theta v(t)$ units of firm shares and rich households are endowed with $(1-\beta\theta)/(1-\beta)$ units of labor and $[(1-\beta\theta)/(1-\beta)]\,v(t)$ units of firm shares, a rich household receives an income stream that is $[(1-\beta\theta)/(1-\beta)]/\theta$ times as large as the one of a poor household.[17] The CES-specification of intertemporal preferences implies that the flow of expenditures of a rich household compared to a poor on the balanced growth path needs to be $[(1-\beta\theta)/(1-\beta)]/\theta$ times as large, too. Recalling that the $mN(t)$ mass producers charge price p_h for the high quality and p_l for the low quality and the $(1-m)\,N(t)$ exclusive producers charge price p_e, the expenditure flow of a poor household is $p_l mN(t)$ and the expenditure flow of a rich household is $[p_h m + p_e(1-m)]\,N(t)$. Hence, the ratio of the expenditure flow of a rich relative to a poor household is

$$\frac{mp_h + (1-m)p_e}{mp_l} = \frac{1-\beta\theta}{(1-\beta)\theta}, \tag{2.5}$$

where $p_e = q_h\mu_R$, $p_l = q_l\mu_P$, and $p_h = q_l\mu_P + (q_h - q_l)\,\mu_R$ (see Proposition 1).

We can now characterize and analyze the balanced growth equilibrium using two equations, a *no-arbitrage curve* and a *resource curve.* Using the no-arbitrage conditions (2.3), we can express the price of the lower quality as

$$p_l = q_l\mu_P = (1-\beta)\left[q_l\mu_R + (q_h\mu_R - a_h)\,G/F\right] + \beta a_l. \tag{2.6}$$

Combining this with the above expression for relative expenditures (2.5) lets us write the price of the exclusive good as

$$p_e = q_h\mu_R = q_h\frac{a_l\beta/(1-\beta) - a_hG/F}{\theta\,(q_h/m - q_l)\,/\,(1-\theta) - q_l - q_hG/F}, \tag{2.7}$$

from which we can infer $p_h = p_l + (q_h - q_l)p_e/q_h$. Plugging (2.7) into the no-arbitrage condition of the exclusive producer and using the Euler equation (2.4) yields the no-arbitrage curve (NA)

$$g = \frac{1}{\sigma}\left(\frac{L}{F}\left[q_h\frac{\beta a_l - (1-\beta)a_hG/F}{\theta\,(q_h/m - q_l)\,/\,(1-\theta) - q_l - q_hG/F} - (1-\beta)a_h\right] - \rho\right), \tag{2.8}$$

which expresses the growth rate g in terms of the fraction of mass goods m. The NA-curve is upward sloping in m if $F\beta a_l > G(1-\beta)a_h$ and downward sloping otherwise. Keeping g constant, the fraction of mass producers m rises in θ, and

[17] Here we stick to the simplifying assumption that the income composition of rich and poor households is identical. As mentioned above, this is a special case that makes the analysis simple and transparent. The more general (and more realistic) case when income composition differs between rich and poor housholds does not add economic substance to the analysis. However, in the next section, when we study transitional dynamics we need to give up this assumption since the wealth distribution is no longer stationary.

falls in β.[18] This is because lower inequality raises the purchasing power of the poor.

A second equation in m and g is derived from the aggregate resource constraint in the economy. Recall that along the balanced growth path the rich consume all $N(t)$ goods in high quality and the poor consume all $M(t)$ mass consumption goods in low quality. Hence, we can write $L = (1-\beta)LN(t)a_h/A(t) + \beta LM(t)a_l/A(t) + \dot{N}(t)F/A(t) + \dot{M}(t)G/A(t)$. Using the equation of motion for the aggregate stock of knowledge (2.1) and the definitions $m = M(t)/N(t)$ and $g = \dot{N}(t)/N(t) = \dot{M}(t)/M(t)$ we can express the resource curve (RC) as

$$g = \frac{L\left[(\psi + (1-\psi)m^{\gamma})^{1/\gamma} - (1-\beta)a_h - \beta a_l m\right]}{F + Gm}. \tag{2.9}$$

Notice that the RC-curve may be upward or downward sloping. On the one hand, there is a demand effect. An increase in m is associated with higher consumption of the poor. Hence, more employment is needed to satisfy this additional demand leaving fewer resources for research. On the other hand, there is a productivity effect. An increase in m means that final output is produced in a more efficient way which saves resources that become available for innovation and growth. Under our specification for the evolution of the knowledge stock (2.1), the productivity effect depends on the importance of process innovation in pushing ahead the knowledge frontier. This is captured by the parameter ψ. The lower is ψ, the more important are process innovations as drivers of technical knowledge and the stronger is the productivity effect. Note also that the distribution parameter θ does not enter the resource curve. The resource curve shifts up when the population share of the poor β rises.

Proposition 1 *A balanced growth equilibrium determined by the intersection of the two curves (2.8) and (2.9) exists if Assumption 1 holds with strict inequalities.*
Proof. *See Appendix B.* ■

The idea of the proof is the following: to determine whether the outcome where mass producers separate households and exclusive producers sell only to rich households is indeed an equilibrium, one needs to compute μ_R and μ_P using the above equations for a given set of parameters, and test whether Assumption 1 holds with strict inequalities. If this is the case, no firm has an incentive to

[18] An increase in θ is offsetting an increase in m as the denominator in the NA-curve is strictly increasing in θ given its derivative with respect to θ of $(q_h/m - q_l)/(1-\theta)^2 > 0$. Similar computations reveal that a decrease in β is offsetting an increase in m.

deviate (see Proposition 1). Assumption 1 holds if the quality gap $q_h - q_l$ is sufficiently high (but not too high) relative to the cost gap $a_h - a_l$ and process innovation costs G; and if inequality is sufficiently high, i.e. the group of poor β is sufficiently large as well as the distribution parameter θ is not too high. Conversely, a low quality gap would induce all firms to become mass producers and supply only the low quality. Similarly, if the quality gap were too high, there would be no incentive to invest in process innovations. These outcomes are less interesting as the model essentially reduces to one of expanding product varieties. When inequality is too low, a further outcome arises in which mass producers sell the low quality to all households. We will characterize these other outcomes in Appendix B in more detail. The existence of a positive growth equilibrium is determined by comparing the horizontal m-axis intercepts of the NA- and RC-curve (denoted by m_{NA} and m_{RC}). Assumption 1 guarantees that the RC-curve (2.9) holds for $g > 0$ if $m = 1$. The equilibrium is unique if $m_{RC} < m_{NA}$ and the NA-curve is upward sloping since the NA-curve is convex and the RC-curve is concave when upward sloping.[19]

2.5 Income Inequality and Technical Change

We will first analyze the equilibrium for the two polar cases of $\psi = 1$ when knowledge spillovers are generated only by product innovations, and of $\psi = 0$ so that technical progress is driven only by process innovations.

2.5.1 Product Innovation as Driver of Productivity Growth

When product innovation is the only driver of productivity growth, we have $\psi = 1$ and equation (2.1) becomes $A(t) = N(t)$. While the no-arbitrage curve (2.8) remains unchanged, the resource constraint simplifies to

$$g = \frac{L\left[1 - (1-\beta)a_h - \beta a_l m\right]}{F + Gm}. \tag{2.10}$$

The resource curve is downward sloping in m, since a larger share of mass producers requires more labor for manufacturing and process innovation, leaving less labor for product R&D, the driver of growth.

[19] The condition $m_{RC} < m_{NA}$ trivially holds if the RC-curve has a vertical axis intercept in the positive (m, g)-quadrant, which is true whenever $\psi^{1/\gamma} > (1-\beta)a_h$. When $m_{RC} \geq m_{NA}$ or the NA-curve is downward sloping, there may (but need not) be multiple balanced growth equilibria. Apart from the locally stable steady state, there exists an intermediate unstable steady state (and a stagnation equilibrium) in that case. See Appendix B.

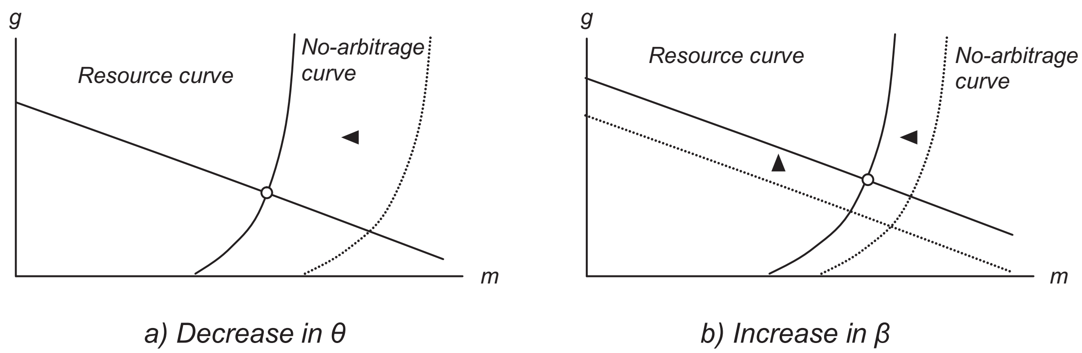

Figure 2.3: Impact of inequality in the case of $\psi = 1$

In the case of $A(t) = N(t)$, inequality is beneficial for growth. A redistribution of income from the poor to the rich (reducing θ) leaves the resource curve unchanged, but shifts the no-arbitrage curve to the left, as depicted in the left-hand panel of Figure 2.3. A richer upper class has a higher willingness to pay for products, and this price effect increases profits. Product inventions become more attractive, spurring technical progress and growth. From a resource point of view, redistributing wealth from the poor to the rich raises exclusion in the economy, setting free resources from the manufacturing and the process R&D sectors, which become available for product R&D, the driver of growth.

Increasing the size of the group of poor households β, while holding θ constant, raises inequality (see section 3.1. above). As can be seen from the right-hand panel of Figure 2.3, the resource curve shifts up and the no-arbitrage curve shifts to the left. The reason is that a higher β is associated with higher inequality. (With θ given, relative incomes of rich households, $(1 - \beta\theta) / [(1 - \beta)\theta]$, increase). While there are less rich households reducing the market for the exclusive goods, the (remaining) rich have a higher willingness to pay. It turns out that the latter (price) effect dominates the former (market size) effect so profits for exclusive producers increase for a given m and g. In the new equilibrium we have fewer mass producers m which releases (manufacturing and process R&D) resources which are channeled into product R&D, and hence growth g is higher.

In sum, higher inequality (either due to a lower θ or due to a higher β, or both) is beneficial for growth, provided that growth is driven purely by product innovations.

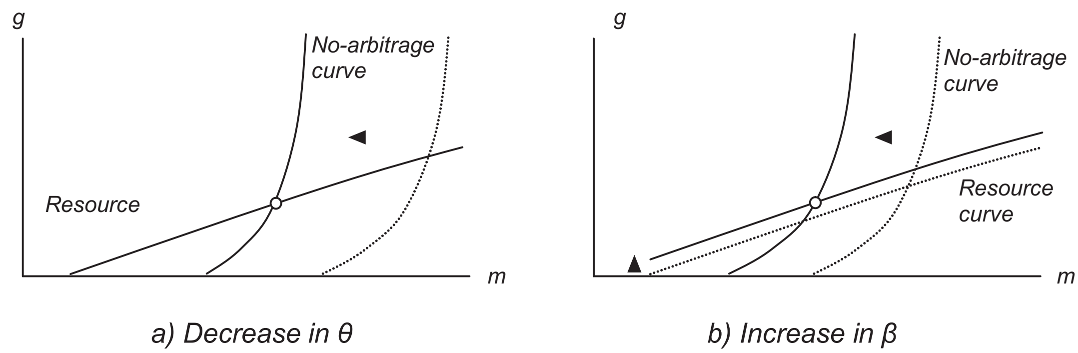

Figure 2.4: Impact of inequality in the case of $\psi = 0$

2.5.2 Process Innovations as Productivity Drivers

The result that inequality is beneficial for growth hinges upon the assumption that only product innovations affect productivity growth whereas process innovation activities do not at all impact technical progress. We now consider the other extreme, when $\psi = 0$, so that technical knowledge is entirely determined by past process R&D activities, $A(t) = M(t)$. The resource curve becomes

$$g = \frac{L\left[1 - (1-\beta)a_h/m - \beta a_l\right]}{F/m + G}, \tag{2.11}$$

and is now upward sloping. As process innovation is the key to become a mass producer, a higher share of mass production m is beneficial for growth. A higher prevalence of mass production raises aggregate productivity. In contast to before, a higher m implies less (low-productive) exclusive sectors which saves resources for process R&D.

A higher extent of inequality due to lower incomes of poor households θ shifts the no-arbitrage curve to the left, as depicted graphically in the left-hand panel of Figure 2.4. The result is less mass production m and also a lower incentive to undertake process innovations. Hence the growth rate g falls.

Increasing the group size of poor households β shifts the no-arbitrage curve to the left and shifts the resource curve up, as shown in the right-hand panel of Figure 2.4. The effect on growth is now ambiguous. When more income is concentrated in the hands of fewer rich, there will be less mass consumption m. This has two effects. On the one hand, the shift from mass consumption to exclusive markets decreases average productivity in manufacturing. On the other hand, less mass production also implies that fewer resources are needed for production which can be used for R&D and growth. Computations show that either effect may dominate.

2.5.3 The General Case

Having analyzed the two polar cases, we have demonstrated that inequality may be either beneficial or harmful for growth, depending on the source of technical progress and productivity growth in the economy. Inequality has an effect on prices and on the size of markets. On the one hand, a higher willingness to pay of the rich households raises prices and profit margins, spurring entry and thus product innovation. On the other hand, a high level of exclusion reduces mass consumption markets, and thus incentives for process innovation.

The general case lies in between the two polar cases. Let us write down the resource curve here as a function of m,

$$g(m) = \frac{L\left[(\psi + (1-\psi)m^{\gamma})^{1/\gamma} - (1-\beta)a_h - \beta a_l m\right]}{F + Gm},$$

The inequality-growth relationship depends on the slope of this function:

Proposition 2 *Given Assumption 1, an increase in inequality due to a lower relative income of poor consumers (lower θ) leads to a lower prevalence of mass producers m. If the resource curve is (locally) decreasing in m, $g'(m) < 0$, inequality raises growth. If it is increasing, inequality hurts growth.*

We have shown above that for a given g, the fraction of mass producers increases in θ. Hence m declines in inequality (given that θ does not enter the resource curve directly), and the impact on growth depends on the slope of the resource curve. In the cases of $\psi = 1$ and $\psi = 0$, we have shown that the resource curve is (globally) downward and upward sloping, respectively. For intermediate cases of ψ, where productivity growth is driven by both product and process innovation, the sign of the inequality-growth relationship depends on the dominating source of technical change and on the extent of inequality. Under the assumption that the aggregate stock of knowledge evolves according to (2.1), the marginal contribution of process innovations, $\partial A(t)/\partial M(t)$ is infinite at $m = 0$, $\lim_{m->0} g'(m) = +\infty$. Hence, as long as $\psi < 1$, the RC curve slopes upwards for low m. For larger values of m the resource curve eventually becomes downward sloping. Intuitively, there are complementarities between product and process innovation. When an economy has invested relatively little in process innovation, it is likely to benefit more from process innovations and vice versa.

Taken together, for $0 < \psi < 1$, the resource curve becomes hump-shaped as depicted in Figure 2.5. Higher inequality fosters growth if inequality is initially low (and the fraction of mass producers is high), whereas higher inequality slows down growth if the extent of inequality is already high initially. Therefore, in a very unequal society that is dominated by exclusive markets lowering inequality is likely to increase growth. The expansion of mass consumption markets spurs process innovation and increases growth. However, in a very egalitarian society, the relationship may be reversed, when innovation incentives are based on a better funded upper class, so that the introduction of new goods becomes more attractive. As a result, both very high levels and very low levels of inequality are harmful for growth. High long-run growth rates are reached by intermediate degrees of inequality.

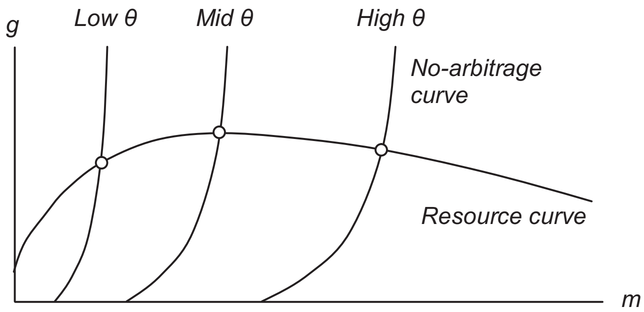

Figure 2.5: Impact of inequality in the case of $\psi \in (0, 1)$

As noted above, both R&D sectors benefit equally from spillovers in our basic setup. Spillovers could affect product and process innovations differently. Suppose for example that there are only spillovers within (but not across) sectors such that $\tilde{F}(t) = F/N(t)$, $\tilde{a}_h(t) = a_h/N(t)$, $\tilde{G}(t) = G/M(t)$, and $\tilde{a}_l(t) = a_l/M(t)$. It is straightforward to verify from the resource curve that growth does no longer depend on inequality in this case,

$$g = \frac{L\left[1 - (1-\beta)a_h - \beta a_l\right]}{F + G}.$$

However, as soon as there are some spillovers from one R&D sector to the other, the inequality-growth relationship depends on the relative strength of product versus process innovation in expanding the technological frontier. In the next section we will analyze an alternative technological specification with learning-by-doing instead of intentional process innovations. We will see that the inequality-growth relationship depends on the relative importance of learning-by-doing effects. This analysis will demonstrate that our basic results and intuitions do not rely on the specific way of modelling the externalities between product and process innovations.

2.6 Product Cycles and Learning-by-doing

Given the symmetry in preferences and technology in our model, firms are indifferent about the timing of process innovation as discussed in the section on R&D. The individual product cycle is indeterminate. There are two natural extensions to our model, either adjusting preferences or technology, which break this symmetry and thus replicate the empirically observed product cycles. A *first* extension models asymmetry into the technology of firms by introducing learning-by-doing at the level of the individual firm. In the *second* extension we relax the symmetry in preferences by introducing hierarchic preferences, a fixed ranking of all varieties in the product space (by attaching unequal utility weights to the various goods).

2.6.1 Learning-by-doing

Process innovation costs $G(j,t)$ may differ across firms and decrease with individual manufacturing experience. Instead of modelling process innovation as an intentional investment of $G(j,t)$ depending on manufacturing experience, it is instructive to analyze the case of process innovation as a pure (passive) by-product of manufacturing,

$$a(j,t) = (1 - \Lambda(j,t))a/N(t),\ \Lambda(j,t) = \int_{-\infty}^{t} \delta x(j,s)\exp(-\delta(t-s))ds,$$

where δ is the speed of learning as well as the depreciation rate of learning capital, and $a(j,t)$ and $x(j,s)$ productivity and production level of firm j (see Matsuyama, 2002). For simplicity, let us assume that there is only one quality level, $q = 1$. Individual productivity of a firm increases due to individual cumulative manufacturing experience, as well as through spillovers from product innovation. In equilibrium, mass consumption markets are more attractive for higher productivity levels due to market size effects. Hence, firms start out exclusively producing for rich households, and eventually become producers for the mass markets, after a fixed time interval Δ:

$$\max_{\Delta} \int_0^{\Delta} (1-\beta)L\left[p_h - (1-\Lambda(j,t))\right]\exp(-rt)dt + \int_{\Delta}^{\infty} L\left[p_l - (1-\Lambda(j,t))\right]\exp(-rt)dt = F/a,$$

where p_h is the price charged by "exclusive producers", and p_l by "mass producers", and we set $w(t) = N(t)/a$ (numéraire). The maximized present value needs to be equal to set-up costs, $\tilde{F}(t)w(t) = F/a$ (given spillovers $\tilde{F}(t) = F/N(t)$), generating a no-arbitrage condition. The optimal period of time Δ for being an exclusive producer is determined by $p_l = (1-\beta)p_h + \beta\left[1 - L(1-\beta)\left[1-\exp(-\delta\Delta)\right] - \delta L/(r+\delta)\right]$,[20] and the fraction of mass producers by Δ:

$$m = 1 - \int_{-\Delta}^{0} gN(0)\exp(gt)dt/N(0) = \exp(-g\Delta).$$

The equilibrium can be analyzed by combining these equations with the Euler equation (2.4) and the relative budget constraint, which in this case is $\xi(m) = \left((1-m)p_h + mp_l\right)/mp_l$, to form a no-arbitrage curve in m and g, as above. The resource curve is determined by the resource constraint:

$$L = gF + \frac{aL}{N(t)}\left[\int_0^{mN(t)} (1-\Lambda(j,t))dj + (1-\beta)\int_{mN(t)}^{N(t)} (1-\Lambda(j,t))dj\right].$$

Our main conclusions remain unchanged. Computations show that the resource curve may be rising or falling in m, depending on the strength of learning-by-doing (LBD). An increase in inequality, through a fall in θ, raises prices and decreases mass consumption markets m, which tends to reduce resources required in manufacturing. However, by lowering aggregate manufacturing, LBD in the economy is reduced. Either effect may dominate. Inequality and exclusion lowers mass production and LBD. If LBD is the dominant driver of productivity growth in the economy, inequality hurts growth.

The LBD formulation provides another demonstration of the close linkage of process innovation and mass consumption markets. Process innovation may be critical *ex-ante* to tap mass consumption markets, and is facilitated *ex-post* by mass production through LBD. Tapping the mass consumption market creates incentives for process innovation, while LBD, in turn, creates incentives to pursue mass market strategies. Note that with the LBD formulation one does not need spillovers from process on product innovations to generate a negative relationship between inequality and growth. As soon as LBD effects are sufficiently

[20] Use Leibniz rule and the fact that $\Lambda(j,t) = L(1-\beta)\left[1-\exp(-\delta t)\right]$ if $t \leq \Delta t$, and $\Lambda(j,t) = L\left[1-\exp(-\delta t)\right] - \beta L\left[\exp(-\delta(t-\Delta t) - \exp(-\delta t)\right]$ if $t > \Delta t$.

strong, inequality lowers growth through reduced mass consumption markets. Finally, one could also use the LBD formulation to model continuous improvements in quality instead of cost-cutting. If experience in manufacturing enables firms to produce higher quality levels instead of saving labor, inequality would be beneficial for product innovation but harmful for quality improvements given lower manufacturing levels.

2.6.2 Hierarchic Preferences

Suppose there is a *hierarchy of needs* as opposed to the symmetric preferences of the main model. Certain more *basic* goods have priority:

$$u(t) = \int_0^{N(t)} \xi(j)x(j,t)q(j,t)dj,$$

where we have added a *hierarchy weight* $\xi(j)$ to felicity which is strictly monotonically decreasing in j. Hence low-j goods get a higher weight than high-j goods, and thus households have a higher willingness to pay for low-j than for high-j goods. Product innovation R&D would focus on the lowest-j goods not yet invented. For balanced growth, the hierarchy weight needs to be a power function, $\xi(j) = j^{-\eta}$ (see Bertola, Foellmi, and Zweimüller, 2006, Chapter 12). The process innovation timing problem becomes:

$$\begin{aligned}
\max_{\Delta} V(j,t) &= \int_t^{t+\Delta} \pi_e(j,s)\exp(-rs)ds + \int_{t+\Delta}^{\infty} \pi_m(j,s)\exp(-rs)ds - G\exp(-r\Delta), \\
\pi_e(j,s) &= L\,(1-\beta)\left[j^{-\eta}q_h\mu_R(s) - a_h\right], \\
\pi_m(j,s) &= L\left[\beta\left(j^{-\eta}q_l\mu_P(s) - a_l\right) + (1-\beta)\left(j^{-\eta}\left((q_h - q_l)\,\mu_R(s) + q_l\mu_P(s)\right) - a_h\right)\right].
\end{aligned}$$

Profit flows depend on hierarchy levels and on time, as $\mu_R(t)$ and $\mu_P(t)$ are increasing at rate ηg.[21] Hence, the difference between profit flows from mass and exclusive strategies grows.[22] In equilibrium, firms start out being exclusive producers. As the difference narrows to $\pi_m(j,s) - \pi_e(j,s) = G$, it becomes optimal for firms to switch to mass strategies, Δ units of time after product innovation (using Leibniz rule). The size makes low-j goods more attractive to sell in mass consumption markets than high-j goods. Note that if we let $\eta \to 0$,

[21] In order that the no-arbitrage condition holds, the initial present value of every newly set up firm must equal F. Hence the hierarchy-independent part of the willingness-to-pay $\mu_i(t)$ must rise at $-\partial/\partial t\left(j^{-\eta}\right) = \eta g$ over time, in order that the overall willingness to pay for a good only depends on the time span since inception, and not on time.

[22] The revenues of mass producers must be higher in equilibrium. Otherwise, firms would never switch to mass strategies given process innovation costs. Note also that both revenue streams grow at the same rate. It follows that $\pi_m(j,s) - \pi_e(j,s)$ grows over time.

the hierarchic preferences formulation converges to the symmetric case of the main text but with a determinate product cycle. If η is sufficiently high, instead, the "innovate-and-wait" pattern arises (studied in Foellmi and Zweimüller, 2006) where firms would innovate early to secure a patent on a low-j good and wait a certain period of time before actually manufacturing the good as demand first needs to mature sufficiently (initially in high quality, supplying it only to the rich).

Hierarchic preferences generate a product cycle where firms initially sell goods exclusively to rich households given their high willingness to pay for new goods even if they are low on their priority list. After a certain period of time, firms invest in process innovation to tap mass consumption markets as their goods have climbed the *relative* hierarchic ladder, being transformed from luxuries into necessities.

2.7 Transitional Dynamics

In our basic framework, both demand and supply shocks may trigger periods of industrial change in which a series of process innovations increases production and access to consumption markets, causing as Perkin (1969) put it *"a revolution in men's access to means of life"* (cited by Mokyr, 1999). In this section, we undertake two thought experiments. In both cases we assume that the economy is initially in an equilibrium that is characterized by low growth and low (or complete absence of) mass production, and analyze exogenous shocks triggering a process of transition toward a new steady state. In doing so, our analysis sheds light on the process by which demand and/or supply shocks generate a take-up of productivity growth and a transition of a society with high exclusion and low consumer-participation of the lower classes to a mass consumption society.

The first thought experiment is a demand shock generated by a major drop in inequality through an increase in θ. Assume that the economy is initially in a steady state characterized by high inequality and low mass production so that the initial balanced growth equilibrium is located on the upward sloping branch of the resource curve (see Figure 2.5). As we have seen in the last section, starting from such an equilibrium, a major drop in inequality leads to a new balanced growth path with higher growth and a higher extent of mass production. One potentially relevant situation from recent economic history is the substantial drop in inequality during the Great Depression and WWII that might help explain the boom in consumer durables in the U.S. of the post-war era. The second thought experiment relates to a positive productivity shock lowering the costs of process innovation, G. Such a shock may trigger an industrial revolution through which an initially stagnant economy of craftsmanship and high exclusion is transformed into an industrialized society with high consumer-participation and growth.

The two state variables that characterize the transition process are the total number of firms $N(t)$ and the number of mass producers $M(t)$. It turns out that, when the economy operates along the balanced growth path both variables grow pari passu. When the economy operates off this path, there are either only product innovations or only process innovations but not both. We summarize this result in

Proposition 3 *Suppose Assumption 1 holds and the economy features both product and process innovations. Then the economy is on the balanced growth path.*
Proof. *See Appendix D.* ■

The proposition implies that, when the economy has too few mass producers $M(t)$, the transition process will be characterized by process innovations only. Similarly, if there are too few exclusive producers $N(t) - M(t)$, the transition process will be characterized only by product innovations. Hence, all adjustments in the state variable $m(t) = M(t)/N(t)$ occur by a "bang-bang" rule. We will also see that this implies that the transition from an old to a new steady state will occur in finite time. This is partly driven by the assumption that $A(t)$ is common across product and process innovation so that the relative cost of the two types of innovations never change. A phase in which one engine of growth stops temporarily is not specific to our set-up. See Matsuyama (1999) for another example where in one phase product variety expansion stops, while the economy accumulates physical capital. In our framework, expansion of variety stops while the economy accumulates process innovation. In fact, this transition closely resembles the related work of directed technical change (see Proposition 1 of Acemoglu and Zilibotti, 2001), where only one type of innovation takes place outside the balanced growth equilibrium. Alternatively, Galor and Moav (2004, 2006) have developed models where in the early stages physical capital accumulation was the prime source of growth, while in latter stages human capital emerged as growth engine.

2.7.1 A Major Drop in Inequality

An exogenous (and instantaneous) drop in inequality leads to transitional dynamics in our framework during which the fraction of firms that have invested in process innovation increases. One can think of the introduction of compulsory schooling, increasing relative productivity of the poor, or an extreme event such as a war lowering financial wealth inequality (such as during WWII), leading to such an adjustment.

Initial and Final Balanced Growth Equilibrium

We assume that both in the initial and final balanced growth equilibrium conditions are such that exclusive producers sell (their high quality) only to the rich; and mass producers sell the high quality to rich and the low quality to poor households. In contrast to the analysis of the last section, we need to relax the assumption of identical endowment distributions. This is because the transition process will be characterized by a situation where the two types of households face different incentives to save and hence will accumulate wealth at unequal speed. In other words, in the transition process, the wealth distribution is no longer stationary invalidating the assumption $\theta_\ell = \theta_v = \theta$. Instead we need to account for the fact that $\theta_v(t)$ changes over time. We focus on the case of log-utility, $\sigma = 1$, for simplicity.

The initial and final balanced growth paths are still characterized by the equations from above, (2.3), (2.4), and (2.9). However, since θ_ℓ may not be equal to θ_v, equation (2.5) needs to be adjusted, as relative lifetime incomes of rich households now depend on the factor income distribution, i.e. on wages $w(t)$ and firm values $v(t)$. With a constant interest rate r and a constant growth rate g, the present value of household i's lifetime income (the right-hand-side of the household i's intertemporal budget constraint) equals $w(t)\ell_i/\rho + v_i(t)$. By normalization, the wage is equal to $w(t) = A(t) = N(t)\left(\psi + (1-\psi)m^\gamma\right)^{1/\gamma}$ and, from the zero-profit conditions (2.3), we have $v(t)L = N(t)\left(F + mG\right)$. As the left-hand-side of a household's intertemporal budget constraint is unaffected by the more general specification of the endowment distributions, we can rewrite equation (2.5) as

$$\frac{mp_h + (1-m)p_e}{mp_l} = \xi(m), \tag{2.12}$$

where relative lifetime incomes $\xi(m)$ are now given by

$$\xi(m) \equiv \frac{\rho(1-\beta\theta_v)(F+mG) + (1-\beta\theta_\ell)L\left(\psi + (1-\psi)m^\gamma\right)^{1/\gamma}}{\rho(1-\beta)\theta_v(F+mG) + (1-\beta)\theta_\ell L\left(\psi + (1-\psi)m^\gamma\right)^{1/\gamma}},$$

with $\xi_m(m) > 0$ since $(\theta_v, \theta_\ell) < (1,1)$. Note also that $\xi(m)$ decreases in both θ_v and θ_ℓ.

We can solve this more general case in a similar way as above. First calculate $p_e = q_h \mu_R$ using equation (2.12) and no-arbitrage conditions (2.3). Then plug the resulting expression into the no-arbitrage condition for the exclusive producer to get a new no-arbitrage curve (2.8)

$$g = \frac{L(1-\beta)}{F}\left[q^h \frac{\beta a_l - (1-\beta)a_h G/F}{(q_h/m - q_l)\,/\,(\xi(m)-1) - (1-\beta)\,(q_l + q_h G/F)} - a_h\right] - \rho, \tag{2.13}$$

For a given growth rate g, raising θ_v or θ_ℓ increases m, since $\xi(m)$ is increasing in m as well as decreasing in θ_v and θ_ℓ. Lowering financial wealth or labor income inequality reduces exclusion. Hence, in much the same way as above, the inequality-growth relationship depends on the slope of the resource curve.

Transition

Now consider a mean-preserving spread in the endowment distributions raising incomes of poor households at the expense of the rich, so that $(\theta_v'(t_0), \theta_\ell') > (\theta_v, \theta_\ell)$, in a balanced growth equilibrium at time $t = t_0$. Imagine that the introduction of compulsory schooling increases relative productivity of the poor,[23] $\theta_\ell' > \theta_\ell$, or shares in firms are redistributed (e.g. during a war) from rich to poor, $\theta_v'(t_0) > \theta_v$.

Figure 2.6 illustrates the transitional dynamics triggered by a drop in inequality. As a result of the shift in purchasing power, poor households increase consumption whereas the consumption of rich households initially stagnates. Since the economy has too few mass producers $M(t)$, demand for the mass production technology is high, and all R&D resources are temporarily directed towards process innovation. The economy reaches the new steady state in finite time at $t = t_2$ when product innovations become attractive again. The figure is drawn in such a way that growth is higher in the final state, which is the case if inequality is sufficiently high in the initial state such that the resource curve is upward sloping ($g'(m) < 0$ see Proposition 3). The following proposition characterizes the transition process in detail:

Proposition 4 *Suppose Assumption 1 holds at all times. a) A fall in inequality at date t_0, from (θ_v, θ_ℓ) to $(\theta_v'(t_0), \theta_\ell')$, triggers a transition period of finite*

[23] Strictly speaking, introducing/increasing compulsory schooling leads to a more equal endowment distribution by changing not only the spread but also the mean of the labor endowment distribution. It is straightforward to see that an increase in L increases growth because the model exhibits a scale effect. Here our focus are distributional consequences, hence we consider mean-preserving spreads.

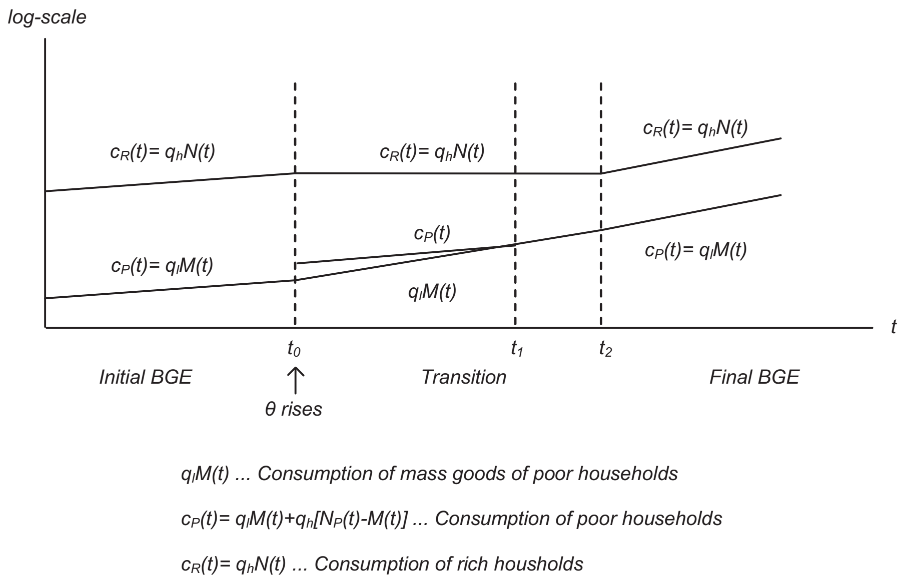

Figure 2.6: Drop in inequality

duration (t_0, t_2) where $\dot{N}(t) = 0$ and $\dot{M}(t) > 0$. A new balanced growth equilibrium with $m' > m$ is reached at date t_2. b) During the entire transition period consumption of the rich stagnates at $c_R(t) = q_h N(t_0)$. c) When the initial reduction in inequality is substantial, $c_P(t)$ jumps to a higher level at date t_0. During a first transition period, $t \in [t_0, t_1)$, $c_P(t) > q_l M(t)$; during a second transition period, $t \in [t_1, t_2)$, $c_P(t) = q_l M(t)$. When the initial reduction in inequality is minor, $c_P(t)$ does not change discontinuously at date t_0, the first transition period does not exist and $c_P(t) = q_l M(t)$ for all $t > t_0$.

See Appendix D for the technical details including a description of the procedure of numerical simulations. If Assumption 1 holds for both (θ_v, θ_ℓ) and $(\theta'_v, \theta'_\ell)$, the balanced growth equilibrium before and after the transition corresponds to a situation where exclusive producers sell (their high quality) only to the rich and the mass producers sell the high quality to the rich and the low quality to the poor. A redistribution from top to bottom has two key effects. *First*, there is an effect on the direction of technical change as only process but no product innovations occur during transition. Redistributing income towards the poor raises their purchasing power and their willingness to pay relative to the one of the rich. Consequently, process innovations become temporarily strictly more attractive than product invention and all R&D activities are concentrated on the implementation of mass production technologies. During this period interest rates are constant and given by

$$r_1 G = \left[\frac{q_l}{q_h} - \frac{a_l}{a_h} \right] L \beta a_h. \tag{2.14}$$

The right-hand side is the incremental profit flow from a mass separating strategy, which must be equal to the current interest rate times the investment for process innovation.[24]

The *second* effect concerns the price setting behavior of exclusive producers. If the drop in inequality is substantial, it becomes attractive for exclusive producers to exploit the higher willingness to pay of the poor. An (endogenous) fraction of exclusive producers will set a price that equals the willingness to pay of the poor and sell temporarily to all households; and the remaining fraction of exclusive producers will still sell only to the rich at a price equal to their (high) willingness to pay.[25] During the first transition period $t \in (t_0, t_1)$ exclusive producers are indifferent between setting a low price and selling to all households and setting a high price and selling only to the rich, i.e. we must have

$$L\left(q_h \mu_P(t) - a_h\right) = L(1-\beta)\left(q_h \mu_R(t) - a_h\right). \tag{2.15}$$

It is also interesting to look at optimal consumption choices during transition. We need to adjust the Euler equation for the rich. Recall that consumption expenditures are $q_h \mu_R(t) N(t)$ and, since in transition $\dot{N}(t)/N(t) = 0$, consumption expenditures grow at rate $\dot{\mu}_R(t)/\mu_R(t)$. The Euler equation therefore determines the growth rate of the willingness to pay of the rich

$$\frac{\dot{\mu}_R(t)}{\mu_R(t)} = r_1 - \rho. \tag{2.16}$$

When the drop in inequality is substantial, poor households' consumption expenditures in the first transition period are $\mu_P(t) c_P(t)$ where $c_P(t)$ is the con-

[24] We have used condition (2.15) to eliminate the willingness-to-pay of rich and poor in the incremental profit flow, $L\left(q_l \mu_P(t) - (1-\beta) q_l \mu_R(t) - \beta a_l\right)$. The flow must be equal to $r_1 G$ since $V_P(t) = G$ and thus $\dot{V}_P(t) = 0$.

[25] The fraction of exclusive producers that sell to all households depends on the extent to which the consumer budget of the poor exceeds the spending on mass consumption goods. In the transition, as the fraction of mass producers increases the share of exclusive producers that sell to all households decreases. By date t_1 the number of firms that have adopted mass production has increased sufficiently so that the optimal spending of the poor exactly coincides with spending on mass consumption goods only.

sumption aggregator for the poor households (see section 3.3). The Euler equation of a poor household therefore is

$$\frac{\dot{\mu}_P(t)}{\mu_P(t)} + \frac{\dot{c}_P(t)}{c_P(t)} = r_1 - \rho. \tag{2.17}$$

Because (2.15) must hold during the first transition phase, it must be that $\mu_P(t)$ increases at a smaller rate than $r_1 - \rho$.[26] Consequently, $\dot{c}_P(t)/c_P(t) > 0$. Denote by $N_P(t)$ the number of goods that the poor can afford. During the first period of transition we have $N_P(t) > M(t)$ and $c_P(t) = q_l M(t) + q_h(N_P(t) - M(t))$. Since $M(t)$ grows faster than $N_P(t)$, there is a date $t = t_1$ where we have reached $M(t_1) = N_P(t_1)$. From date t_1 onwards we have $c_P(t) = q_l M(t)$. The equal-profit condition (2.15) does not hold anymore and exclusive producers are strictly better off selling only to the rich. $\mu_R(t)$ continues to grow at rate $r(t) - \rho$, but $\mu_P(t)$ grows more slowly. Interest rates are no longer constant, but still determined by incremental profit flows and investment costs for process innovation.

The final law of motion comes from the resource constraint. Recalling that in the entire transition period we have $\dot{N}(t) = 0$ and $N(t) = N(t_0)$ we can write

$$\dot{M}(t)G/L = A(N(t_0), M(t)) - \beta\left[M(t)a_l + (N_P(t) - M(t))a_h\right] - (1-\beta)N(t_0)a_h. \tag{2.18}$$

Moreover, we have initial conditions $M(t_0) = mN(t_0)$ and $N(t_0)$, and transversality conditions for rich and poor households. At date t_2, the economy reaches the new balanced growth equilibrium with $m(t) = m'$ in finite time as soon as product innovation becomes attractive again, $r(t)F = L(1-\beta)(q_h\mu_R(t) - a_h)$.

In summary, a substantial drop in inequality may trigger a period of industrial change where innovation activity is purely directed towards process innovation. Such a transition could have been triggered by the substantial drop in inequality during the Great Depression and WWII, helping to explain the boom in consumer durables in the U.S. in the post-war era. Note that in the opposite case of a decrease in (θ_v, θ_ℓ), raising inequality, one can show that innovation is purely directed to product innovation during the transition.

2.7.2 Positive Productivity Shock

Process innovations, such as the introduction of assembly lines, play an important role in the emergence of modern mass consumption markets. In this

[26] >From (2.15) it is straighforward to calculate $\dot{\mu}_P(t)/\mu_P(t) = [(1-\beta)\dot{\mu}_R(t) + \beta a_h] \,/\, [(1-\beta)\mu_R(t) + \beta a_h] < \dot{\mu}_R(t)/\mu_R(t) = r_1 - \rho$

subsection we study an economy in a stagnant/low-growth state where process innovation initially is too expensive or not available at all (G prohibitively high). If a positive productivity shock lowers G sufficiently, the economy experiences a takeoff, transforming a stagnant (or low-growth) highly exclusive economy into an economy with high consumer-participation and growth. One needs the restriction of $\gamma > 0$ in order to have sustained growth in the initial stage without any experience in process innovation (so that product and process innovations are not too complementary).

Initial Exlusive Stage

Suppose that, initially, the economy is characterized by a balanced growth equilibrium where process innovations are absent altogether. More precisely, assume initially G is too high to make process innovations sufficiently attractive. In such a steady state the economy invests only in product innovations. Active firms do not have access to the mass production technology. (Think of the high quality as goods produced by craftsmen. The poor households can only afford a very limited subset of these expensive, hand-crafted goods, e.g. one set of furniture which holds for a lifetime or one tailored suit.) Hence, the initial equilibrium is characterized by a situation where a fraction $n_P = N_P(t)/N(t)$ of producers serve the entire customer base at price $q_h\mu_P$ and a fraction $1 - n_P = (N(t) - N_P(t))/N(t)$ sells their product only to the rich at price $q_h\mu_R$. Lifetime income of houshold i still is $w(t)\ell_i/\rho + v_i(t)$. However, since the (initial) balanced growth equilibrium features $m = 0$, we have $w(t) = A(t) = N(t)\psi^{1/\gamma}$ and $v(t)L = N(t)F$. The relative budget constraint of a rich to a poor household (2.5) now becomes

$$\frac{(1-n_P)\mu_R + n_P\mu_P}{n_P\mu_P} = \xi(0) \equiv \frac{\rho(1-\beta\theta_v)F + (1-\beta\theta_\ell)L\psi^{1/\gamma}}{\rho(1-\beta)\theta_v F + (1-\beta)\theta_\ell L\psi^{1/\gamma}},$$

the no-arbitrage curve reads

$$g = \frac{L(1-\beta)}{F}\left[\frac{\beta a_h}{(1/n_P - 1)\,/\,(\xi(0)-1) - (1-\beta)} - a_h\right] - \rho,$$

and the resource curve

$$g = \frac{L[\psi^{1/\gamma} - (1-\beta+n_P\beta)a_h]}{F}.$$

In this initial stage, the long-run growth performance of the economy is weak because technical progress is only fueled by product R&D whereas process R&D projects are not undertaken at all. As a result manufacturing acitivities and product invention is relatively unproductive (high $\tilde{F}(t)$ and $\tilde{a}_h(t)$). In this initial stage, raising inequality clearly is beneficial for growth as higher exclusion frees up resources for product R&D.

Transition

Consider an exogenous positive productivity shock, $G' < G$, lowering investment costs of process innovations sufficiently such that

$$rG' < \left[\frac{q_l}{q_h} - \frac{a_l}{a_h}\right] L\beta a_h < rG. \tag{2.19}$$

The incremental profit flow of having implemented process innovation must be greater than prevailing interest rates times the investment amount, G'.[27] Process innovations become attractive once productivity gains, a_l/a_h, sufficiently outweigh quality discounts, q_l/q_h. Such a positive supply shock triggers an industrial revolution in which a series of process innovation transforms the initial exclusive society into a modern mass consumption society.

Figure 2.7 displays the evolution of the economy around the transition from an exclusive to a mass consumption society. After the economy experiences a positive productivity shock lowering G at time t_0, product innovation temporarily halts as firms focus on innovating their manufacturing processes. In this phase, consumption of the rich stagnates, whereas the product range of the poor grows as they shift their consumption towards goods at lower prices and quality once available. After all mass producers have innovated their manufacturing process, product invention activities resume once the economy reaches the new balanced growth equilibrium with higher growth and lower exclusion in finite time. The transition process resembles the one following a shift in inequality from above (see Appendix D):

Proposition 5 *Suppose Assumption 1 holds after the shock: a) A substantial drop in process innovation costs, $G' < G$, at $t = t_0$ such that condition (2.19) holds, triggers a transition of finite duration $t \in (t_0, t_2)$ with $\dot{N}(t) = 0$ and $\dot{M}(t) > 0$. From t_2 onwards, the economy is in a new steady state with $m > 0$. b) Consumption of the rich stagnates at $c_R(t) = q_h N(t_0)$ during the entire transition. c) Consumption of the poor jumps to $c_P(t_0) = q_h N_P(t_0)$ at date t_0. During a first phase of the transition, $t \in (t_0, t_1)$, $c_P(t) = q_l M(t) + q_h(N_P(t) - M(t))$*

[27] Similarly to the first example of a transition, we have used condition (2.4) to eliminate the willingness-to-pay of rich and poor in the additional profit flow, $L\left(q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l\right)$, which initially must hold.

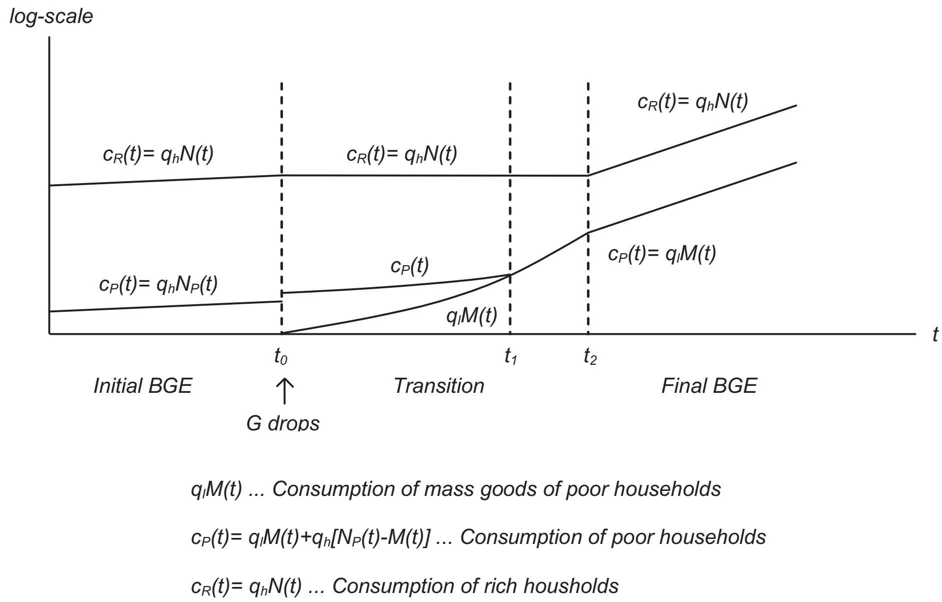

Figure 2.7: Positive productivity shock

grows at a rate lower than $\dot{M}(t)/M(t)$. During a second phase of transition $t \in (t_1, t_2)$, $c_P(t)$ and $M(t)$ grow pari passu.

If poor households immediately stopped consuming higher quality goods, consumption of the poor would need to drop to zero, since immediately after the shock no firm is able to offer the low quality yet. This cannot be the case due to infinite marginal utility at zero consumption. Hence, there is an initial phase with $M(t) < N_P(t)$ corresponding to the initial phase following a drop in inequality, characterized by the dynamic system (2.15)-(2.18) with initial conditions $M(t_0) = 0$ and $N(t_0) > 0$, and transversality conditions. During the first phase $t \in (t_0, t_1)$ poor households purchase both high-quality goods produced with the inefficient technology and low-quality goods produced with the new mass production technology. From date t_1 onwards, only firms that have made the process innovation sell to the poor. In this second transition phase all R&D activity still consists of process innovation and only when the new balanced growth level of $m = M(t)/N(t)$ has been reached, firms start developing new products. Given the stagnant consumption of rich households, $\dot{N}(t) = 0$, prices for exclusive goods increase relative to mass goods until product innovation becomes attractive again, and the economy reaches the new balanced growth equilibrium, corresponding to the one of Proposition 2 with Assumption 1 holding. Growth is higher in the new balanced growth equilibrium if process innovation is sufficiently important for technical progress and productivity growth (if ψ and γ are not too high).

Process innovations are able to transform the initial stagnant/low-growth economy burdened by high exclusion into a modern mass consumption society characterized by significantly higher growth and lower exclusion. Notice that our results are quite different from those in Matsuyama (2002) who also studies the transition to a mass consumption society. In contrast to the learning-by-doing formulation of Matsuyama, where competitive firms experience technical progress due to past production experience, in our case intentional innovation activities drive the adoption of mass production technologies and the introduction of mass consumption goods. Hence, under certain parameter values, our analysis may feature a situation where, in the initial exclusive society, inequality is unambiguously beneficial for growth, while after the transition to a new steady state, the inequality-growth relationship may be turned upside down. Once mass production technologies break even, a more egalitarian society increases mass consumption markets fostering process innovation and brings the economy on a steeper long-run growth path.

2.8 Conclusion

In this essay we presented an endogenous growth model where firms invest both in product and process innovations. Product innovations (that open up completely new product lines) satisfy the luxurious wants of the rich. Subsequent process innovations (that decrease costs per unit of quality) transform the luxurious products of the rich into conveniences of the poor. A prototypical example for such a product cycle is the automobile. Initially an exclusive product for the very rich, the automobile became affordable to the middle class after the introduction of Ford's Model T, the car that "put America on wheels". We argue that recent economic history is full of examples where consumer durables followed a similar product cycle.

Our analysis shows that the extent of economic inequality in a society generates substantially different incentives for product and process innovation. An egalitarian society creates strong incentives to adopt mass production technologies that allow the production of low-quality low-cost versions of existing luxuries (such as Model T). In contrast, an unequal society creates strong incentives for product innovations (new luxuries). Depending on which type of innovative activity drives technical progress, economic inequality is harmful or beneficial for long-run growth. This distinct role of product and process innovations goes in an important way beyond standard R&D based growth models, in which process innovations and product inventions are often mathematically similar (Acemoglu, 2009). To investigate the role of income inequality, one must deviate from the standard homothetic preferences. If the wealthy upper class consumes both more and better goods than the large majority of poorer households, in line with both casual observation and empirical evidence, inequality shapes product markets and thus relative incentives for product versus process innovation.

While our basic framework determines the extent of mass production, individual product cycles are indeterminate. We proposed two natural extensions to get rid of this indeterminacy and incorporate deterministic product cycles in our analysis. A first extension is learning-by-doing where production experience lets production costs fall over time. Goods that are introduced earlier can be produced at lower cost, increasing innovators' incentive to open up mass markets. A second extension are hierarchic preferences where goods can be ranked according to priority in consumption (i.e. yield asymmetric utilities). Product innovations follow the consumption hierarchy in the sense that product R&D expenditures are directed towards (not yet invented) goods with highest priority and process innovations are undertaken when the incomes of the poor have sufficiently increased. Both extensions generate a deterministic cycle with an initial phase of exclusion (only the rich can afford the new product) followed by the phase of mass consumption. Furthermore, our framework is sufficiently simple and tractable so that we can characterize not only balanced growth paths but also transition processes. Studying transitional dynamics is not only interesting from a methodological point of view but is relevant to better understand episodes in recent economic history. For instance, our analysis has shown that a major redistribution of economic resources such as the fall in U.S. income inequality between the Great Depression and WWII may help to explain the post-war boom in consumer durables. Our analysis shows that a demand shock arising from a major income redistribution temporarily generates very strong incentives for process innovations and the introduction of mass consumption goods. Similarly, major technological inventions, such as the assembly line, also give temporary strong incentives to implement mass production technologies so that existing sectors – one after the other – adopt mass production technologies, leading to a trickle-down process from which the poor benefit disproportionately.

For the sake of simplicity and tractability, our model reduced the income distribution to two groups of households. A more general income distribution would smooth the product cycle with penetration levels following logistic Engel curves in the aggregate (rather than a jump as in the stylized case of two groups of consumers). A new producer would start out serving only the richest households and then, by setting lower prices, expand the market step-by-step (in the case of a discrete number of distinct groups) or continuously (in the case of a continuous endowment distribution). Once a certain "cut-off" date has been reached, the producer would invest in process innovation. However, apart from generating more realistic dynamics of product penetration, such a generalization – while substantially complicating the formal analysis – would add little additional economic insight to the model.

As discussed above, our model has abstracted from continuous quality improvements of existing goods. It was assumed that quality adjustments occur only once – when the process innovation is made and the mass production technology together with a low-quality version of an existing luxury good is implemented. However, continuous quality improvements both of luxuries and mass consumption goods are important features of reality. Our model could be easily adapted to account for exogenous quality increases. If q_h and q_l increased at an exogenous rate, all features of our model would remain the same. At a more general level, understanding how the quality upgrading of existing products interacts with the degree of inequality in society is an interesting direction of research and is the topic of the next chapter.

2.9 Acknowledgment

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2.10 Appendix

Appendix A (Chapter 2): Proof of Proposition 1

Taking labor as the numeraire so that $w(t) = A(t)$, we can rewrite marginal costs $w(t)\tilde{a}_k(t) = a_k$ for $k \in \{l, h\}$ given spillovers. A mass producer selling the high quality to the rich and the low quality to the poor faces the following profit maximization problem:

$$\max_{p_h, p_l} \left[L(1-\beta)(p_h - a_h) + L\beta(p_l - a_l) \right],$$

s.t. (i) $p_h \le q_h \mu_R$, (ii) $p_l \le q_l \mu_P$, (iii) $q_h \mu_R - p_h \ge q_l \mu_R - p_l$, and (iv) $q_l \mu_P - p_l \ge q_h \mu_P - p_h$,

The constraints are based on the first-order conditions of households (2.2). (i) and (ii) ensure that households purchase the good (rationality constraints), and (iii) and (iv) ensure that rich households prefer to buy the high quality and poor the low (incentive constraints). Notice that a firm cannot separate the rich into the low quality and the poor into the high given the higher willingness to pay of the rich, $\mu_R > \mu_P$.[28]

Constraint (iii) and $\mu_R > \mu_P$ imply $q_h \mu_R - p_h \ge q_l \mu_R - p_l > q_l \mu_P - p_l$. Hence if constraint (ii) were inactive, so would be (i). But then the firm could increase both prices by the same amount without violating (iii) and (iv). Hence constraint (ii) must be active, $q_h \mu_R - p_h \ge q_l \mu_R - p_l > q_l \mu_P - p_l = 0$, which implies that constraint (iii) must be active, too. Otherwise the firm could increase the price of the high quality without violating constraints (iii) and (i). Since constraint (iii) is active, $q_h \mu_R - p_h = q_l \mu_R - p_l > q_l \mu_P - p_l = 0$, constraint (i) cannot be active. Rewritting the active constraint (iii), $p_h - p_l = q_h \mu_R - q_l \mu_R > q_h \mu_P - q_l \mu_P$ shows that constraint (iv) is not active as well. Hence constaints (ii) and (iii) are active, $p_l = q_l \mu_P$ and $q_h \mu_R - p_h = q_l \mu_R - p_l$, and a separating mass producer optimally sets prices $p_l = q_l \mu_P$ and $p_h = q_l \mu_P + (q_h - q_l)\mu_R$.

Recall that a mass producer has four other options besides separating the rich into the high quality and the poor into the low (h, l): sell the high quality only to rich $(h, 0)$ or to all households (h, h), or sell the low quality only to rich $(l, 0)$ or to all households (l, l). The five options yield the following profit flows:

$$\begin{aligned}
\pi_{h,0} &= L(1-\beta)(q_h \mu_R - a_h), \\
\pi_{h,h} &= L(q_h \mu_P - a_h), \\
\pi_{h,l} &= L\beta(q_l \mu_P - a_l) + L(1-\beta)((q_h - q_l)\mu_R + q_l \mu_P - a_h), \qquad (2.20) \\
\pi_{l,l} &= L(q_l \mu_P - a_l), \\
\pi_{l,0} &= L(1-\beta)(q_l \mu_R - a_l).
\end{aligned}$$

It is easy to verify that if Assumption 1 holds, separating households (h, l) is an optimal strategy for mass producers. Condition (i) $(q_h - q_l)\, \mu_R > a_h - a_l$ ensures

[28] Incentive constraints of $q_l \mu_R - p_l \ge q_h \mu_R - p_h$ and $q_h \mu_P - p_h \ge q_l \mu_P - p_l$ would require $(q_h - q_l)\mu_P \ge p_h - p_l \ge (q_h - q_l)\mu_R$, which cannot hold.

that selling the low quality to all households (l,l) yields lower profits. Condition (iii) $q_l\mu_P - (1-\beta)\,q_l\mu_R - \beta a_l \geq 0$ ensures that selling only the high quality to rich households $(h,0)$ yields equal or lower profits. And since condition (ii) $(1-\beta)\,(q_h\mu_R - a_h) \geq (q_h\mu_P - a_h)$ ensures that exclusive producers (weakly) prefer selling the high quality only to rich households instead to all, selling the high quality to all households (h,h) must generate lower profits for mass producers, as well. And finally, condition (i) also ensures that selling the low quality only to rich households $(l,0)$ is inferior (to selling the high quality only to rich households and thus to separating households). Similarly for exclusive producers which can only supply the high quality, condition (ii) ensures that selling only to rich households is an optimal strategy.

If conditions (ii) and (iii) in Assumption 1 hold with strict inequality, exclusive producers sell only to the rich generating $\pi_e = \pi_{h,0}$, and mass producers separate households generating $\pi_m = \pi_{h,l}$, proofing part (a) of Proposition 1. When condition (ii) holds with equality, exclusive firms are indifferent between selling only to rich and to all households, $\pi_e = \pi_{h,0} = \pi_{h,h}$, proofing part (b). And when condition (iii) holds with equality, mass producers are indifferent between selling only to rich and selling to all, separating households, $\pi_m = \pi_e$, proofing part (c).

If Assumption 1 does not hold, it might be more profitable for exclusive producers to sell the high quality to all households and/or for mass producers to sell only one quality either only to rich or to all households. Appendix B takes into account the general equilibrium to say more about the different outcomes.

Appendix B (Chapter 2): Proof of Proposition 2

In a first step we prove the following lemma stating the possible equilibrium outcomes:

Lemma 1 *In a balanced growth equilibrium, four outcomes are possible: (1) some firms sell the high quality only to rich while others the high quality to rich and the low quality to poor, (2) some firms sell the high quality only to rich while others the low quality to all, (3) all firms only sell the high quality, some only to rich while others to all, and (4) all firms only sell the low quality, some only to rich while others to all.*

Proof. In any equilibrium, among the five options of firms (see Appendix A), two are equilibrium strategies: some firms sell to all households since otherwise poor households would consume nothing, and some firms sell only to rich households.[29] This leaves six combinations of two strategies of which two can be ruled out:

Compare profit flows (equations 2.20) to see that "$(q_h - q_l)\mu_P > a_h - a_l$" $\Longrightarrow$ "$\pi_{h,h} > \pi_{l,l}$", "$(q_h - q_l)\mu_R > a_h - a_l$" $\Longrightarrow$ "$\pi_{h,0} > \pi_{l,0}$" and "$(q_h - q_l)\mu_R > a_h - a_l$" $\Longrightarrow$ "$\pi_{h,l} > \pi_{l,l}$". Since $\mu_R > \mu_P$, we have "$\pi_{h,h} \geq \pi_{l,l}$" $\Longrightarrow$ "$\pi_{h,0} > \pi_{l,0}$", "$\pi_{l,0} \geq \pi_{h,0}$" $\Longrightarrow$ "$\pi_{l,l} > \pi_{h,h}$", and "$\pi_{h,0} > \pi_{l,0}$" $\Longleftrightarrow$ "$\pi_{h,l} > \pi_{l,l}$". Hence we can rule out outcomes where some firms separate and other firms sell the low quality only to rich households. We can also rule out outcomes where some firms sell the high quality to all households and other firms the low quality only to rich households, which require $\pi_{l,0} - \pi_{h,0} \geq \pi_{l,l} - \pi_{h,h}$, implying $(q_h - q_l)\mu_P\,[1/\beta - (1-\beta)\mu_P/\beta/\mu_R] \geq a_h - a_l$. But from above we know that "$\pi_{l,0} \geq \pi_{h,0}$" $\Longrightarrow$ "$\pi_{l,l} > \pi_{h,h}$", implying $(q_h - q_l)\mu_P < a_h - a_l$, which contradicts the inequality in the previous sentence, since the term in the square bracket is smaller than one ($\mu_R > \mu_P$, and $\beta < 1$). ■

Note that in any balanced growth equilibrium one of the four outcomes prevails. On transitional equilibrium paths we have shown that, if $m(t)$ is too low, three strategies co-exist for general parameter values. Focusing on balanced growth paths, let us characterize these four outcomes in more detail starting with the one of the main text proofing Proposition 2.

A balanced growth equilibrium determined by (2.8) and (2.9) exists if Assumption 1 holds with strict inequalities. From Proposition 1 we know that exclusive producers sell only to the rich and mass producers separate households if Assumption 1 holds with strict inequalities. Hence one needs to compute μ_R and μ_P for a given set of parameters, using equations (2.8) and (2.9) and the expressions for prices of Section 4.3, and test whether Assumption 1 holds. If this is the case, no firm has an incentive to deviate, and the outcome is indeed an equilibrium.[30] Computations have shown that Assumption 1 holds in a balanced growth equilibrium if the quality gap $q_h - q_l$ is sufficiently high relative to the cost gap $a_h - a_l$ and process innovation costs G; and if inequality is sufficiently high, i.e. the group of poor β is sufficiently large as well as θ not too high. If Assumption 1 is violated, alternative outcomes prevail (see below).

[29] Firms which sell to all households cannot charge the entire willingness to pay of the rich (even when separating households they need to leave an "informational rent" to incentivize rich households to buy the high quality). Hence rich would have no binding first-order condition (i.e. would not exhaust their budgets) if all firms sold to all households. We can rule out such equilibrium outcomes, since rich households would have an infinite willingness to pay and thus firms would have an incentive to sell only to rich households instead.

[30] Furthermore, one can show that the equilibrium is unique by checking that firms have incentives to deviate in every alternative equilibrium outcome (see below).

In order to determine existence of a positive growth equilibrium, denote the horizontal m-axis intercepts of the NA- and RC-curve as m_{NA} and m_{RC}. If $m_{RC} < m_{NA}$ a positive balanced growth equilibrium must exist. The left hand side of (2.9) is increasing in ψ for $m < 1$. Hence, the RC-curve shifts downwards when ψ decreases. Thus, $m_{RC}|_{\psi>0} < m_{RC}|_{\psi=0} = (1-\beta)a_h/(1-\beta a_l)$, by using (2.9). Since $(1-\beta)a_h/(1-\beta a_l) < 1$ (otherwise RC and NA could not cross at $m < 1$ for $\psi = 0$ thereby violating assumption 1), the RC-curve (2.9) is fulfilled for $g > 0$ if $m = 1$. We derive a sufficient condition for $m_{RC} < m_{NA}$

$$(1-\beta)a_h/(1-\beta a_l) <$$
$$\theta\left(q_l/q_h + (1-\theta)G/F + (1-\theta)\left[\beta a_l - (1-\beta)a_h G/F\right]/\left[\rho F/L + (1-\beta)a_h\right]\right)^{-1} = m_{NA}.$$

Note further that the condition $m_{RC} < m_{NA}$ trivially holds if the RC-curve has a vertical axis intercept in the positive (m, g)-quadrant, which is true whenever $\psi^{1/\gamma} > (1-\beta)a_h$.

The balanced growth equilibrium is necessarily unique if the NA-curve is upward sloping (which holds true if $a_l\beta/(1-\beta) > a_h G/F$). The NA-curve is always convex in m. To see this, note that

$$\partial^2 q_h\mu_R/\partial m^2 = 2\zeta(1-\theta)\left[a_l\beta/(1-\beta) - a_h G/F\right]\left[\theta/m - \zeta\right]^{-3} > 0$$

with $\zeta \equiv q_l/q_h + (1-\theta)G/F$. The definition of (2.7) requires that the nominator and the denominator have the same sign such that $q_h\mu_R > 0$. The RC-curve is concave when it is upward sloping, this holds true as $(\psi + (1-\psi)m^\gamma)^{1/\gamma}$ is a concave function. Hence, the curves can cross only once in only once in the positive (m, g)-quadrant as long as the horizontal m-axis intercept of an upward sloping NA-curve lies to the right of the RC-curve. For $m_{RC} \geq m_{NA}$ or a downward sloping NA-curve, a positive growth equilibrium exists as well but it is not necessarily unique.

Appendix C (Chapter 2): Alternative Equilibrium Outcomes

When one of the conditions in Assumption 1 is violated, alternative outcomes will arise. We briefly discuss these outcomes. *First*, mass producers may supply only the low quality to both poor and rich households while exclusive producers sell only to the rich. Along the lines of the main text, we can derive prices, a no-arbitrage curve and a resource curve, respectively, for such an outcome,

$$
\begin{aligned}
p_l &= q_l \mu_P = (1 + G/F)(1-\beta)\left(q_h \mu_R - a_h\right) + a_l, \\
p_e &= q_h \mu_R = \frac{a_l - (1+G/F)(1-\beta)a_h}{(1/m - 1)\,/\,(\xi(m) - 1) - (1-\beta)(1+G/F)}, \\
g &= \frac{L(1-\beta)}{F}\left[\frac{a_l - (1+G/F)(1-\beta)a_h}{(1/m - 1)\,/\,(\xi(m) - 1) - (1-\beta)(1+G/F)} - a_h\right] - \rho, \\
g &= \frac{L\left[\left(\psi + (1-\psi)m^{\gamma}\right)^{1/\gamma} - (1-m)(1-\beta)a_h - m a_l\right]}{F + Gm}.
\end{aligned}
$$

This outcome is qualitatively similar to the one we focus on with one difference: Even in the case of $A(t) = N(t)$, i.e. $\psi = 1$, inequality may be harmful for growth. For a sufficiently small a_l, the resource curve may be increasing in m. An increase in the fraction of mass producers m may set free resources for product R&D, as mass producers only use the less laborious process. Even though more goods are produced, less production labor is needed. If a_l is not sufficiently small, the results are analogous to the main text. The conditions for this case are that exclusive producers prefer selling only to rich, $(1-\beta)(q_h\mu_R - a_h) > (q_h\mu_P - a_h)$, and mass producers prefer selling the low quality to all households, $a_h - a_l > (q_h - q_l)\mu_R$ and $(q_l\mu_P - a_l) > (1-\beta)(q_l\mu_R - a_l)$.

Second, if process innovation costs are too high, no firm invests in mass production and firms either sell to rich or to all households. Such an outcome corresponds to the initial stage in Section 6.2, and the equilibrium is characterized by the equations presented there. Recall that in this equilibrium outcome, inequality unambiguously is beneficial for growth as the resource curve is downward sloping in n_P given the absence of process innovation. Process innovation costs are too high if $G > \max(\pi_{h,l}/\,(g+\rho) - F,\ \pi_{l,l}/\,(g+\rho) - F,\ \pi_{l,0}/\,(g+\rho) - F)$.

Third, the last outcome arises in the opposite case where the mass production technology is too attractive. The resulting outcome is qualitatively equivalent to the previous one (initial stage in Section 6.2), substituting $(F+G, a_l, q_l, \xi(0), 1)$ for $\left(F, a_h, q_h, \xi(1), \psi^{1/\gamma}\right)$, and arises if $F > \max(\pi_{h,0}/\,(g+\rho)\,, \pi_{h,h}/\,(g+\rho))$ and $a_h - a_l > (q_h - q_l)\mu_R$, that is if process innovation costs G are sufficiently low, and the quality gap $q_h - q_l$ relatively low compared to the cost gap $a_h - a_l$. Also in this equilibrium outcome, inequality is unambiguously beneficial for growth as the resource curve is downward sloping since all firms, even the one only selling to rich households, invest in the low-quality process innovation, $m = 1$.

Appendix D (Chapter 2): Transitional Dynamics

When the economy operates off the balanced growth path, there are either only product innovations or only process innovations but not both:

Proof of Proposition 4: Suppose the economy is in an equilibrium but not necessary the steady state where both product and process innovation occur. Since $V_N(t) = F$ and $V_M(t) = G$ hold, the instantaneous interest rate is given by

$$r(t) = L\left[q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l\right]/G = (1-\beta)L(q_h\mu_R(t) - a_h)/F. \tag{2.21}$$

The Euler equations of rich and poor, and the resource constraint read

$$\begin{aligned}\dot{\mu}_R(t)/\mu_R(t) &= r(t) - \rho - \dot{N}(t)/N(t), \quad \dot{\mu}_P(t)/\mu_P(t) = r(t) - \rho - \dot{M}(t)/M(t),\\ \dot{M}(t)G + \dot{N}(t)F &= L\left(\psi N(t)^\gamma + (1-\psi)M(t)^\gamma\right)^{1/\gamma} - L\beta M(t)a_l - L(1-\beta)N(t)a_h.\end{aligned}$$

We reduce this system of differential equations to get a single equation in $\mu_R(t)$ and $M(t)/N(t)$. Rewrite the resource constraint

$$\frac{\dot{M}(t)}{M(t)}\frac{M(t)}{N(t)}G + \frac{\dot{N}(t)}{N(t)}F = L\left(\psi + (1-\psi)\left(\frac{M(t)}{N(t)}\right)^\gamma\right)^{1/\gamma} - L\beta\frac{M(t)}{N(t)}a_l - L(1-\beta)a_h \equiv \phi\left(\frac{M(t)}{N(t)}\right)$$

Rearranging (2.21) we get $q_l\mu_P(t) = (1-\beta)\left(q_l + q_hG/F\right)\mu_R(t) + \beta a_l - (1-\beta)a_hG/F$. We take the derivative and insert this into the Euler equation of the poor to get

$$\frac{\dot{\mu}_R(t)}{\mu_R(t) + \left[\beta a_l - (1-\beta)a_hG/F\right]/\left[(1-\beta)\left(q_l + q_hG/F\right)\right]} = \\ (1-\beta)\frac{L}{F}(q_h\mu_R(t) - a_h) - \rho - \left(\frac{M(t)}{N(t)}G\right)^{-1}\left(\phi\left(\frac{M(t)}{N(t)}\right) - \frac{\dot{N}(t)}{N(t)}F\right),$$

and use the Euler equation of the rich to form

$$\frac{\dot{\mu}_R(t)}{\mu_R(t) + \left[\beta a_l - (1-\beta)a_hG/F\right]/\left[(1-\beta)\left(q_l + q_hG/F\right)\right]} + \frac{\dot{\mu}_R(t)}{\mu_R(t)}\frac{F}{GM(t)/N(t)} = \\ (1-\beta)\frac{L}{F}(q_h\mu_R(t) - a_h) - \rho - \left(\frac{M(t)}{N(t)}G\right)^{-1}\left(\phi\left(\frac{M(t)}{N(t)}\right) - (1-\beta)L(q_h\mu_R(t) - a_h) + \rho F\right).$$

We see that $\dot{\mu}_R(t)$ is monotonically increasing in $\mu_R(t)$. Denote the steady state level of $\mu_R(t)$ by μ_R^{SS}. Therefore, if $\mu_R(t) > (<)\mu_R^{SS}$, $\mu_R(t)$ will grow (fall) without bound. Hence, there is only one equilibrium: $\mu_R(t)$ must immediately adjust to μ_R^{SS}. As $\mu_P(t)$ and $\mu_R(t)$ are monotonically related through (2.21) the analoguous holds true for $\mu_P(t)$ as well. We conclude that in the presence of both process and product innovations the economy is in steady state.